# THE INTERSECTION OF PROBLEM POSING AND CREATIVITY: A REVIEW

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***Abstract.*** *In this article, we take an in-depth look at research on the intersection of problem posing and creativity in order to present its current state of research in a systematic review. A full search in top journals from mathematics education and the Web of Science revealed only 15 articles from different genres, of which 11 were included in the analysis. Those articles were sorted into two clusters, depending on whether the articles focus on the identification or the fostering of creativity.*

*Key words: Problem Posing, Creativity, Review*

## INTRODUCTION

In the 1990's, Edward Silver published two seminal articles in which he addressed both mathematical problem posing and mathematical creativity. The first article (Silver, 1994) deals with problem posing, emphasizing it as a characteristic of creative activities and mathematical ability. In the second article, Silver (1997) takes the opposite perspective, mainly addressing creativity and highlighting its connections to problem posing (as well as problem solving). Both contributions are widely cited in research literature and constitute the theoretical foundation for many studies dealing in one way or another with problem posing and creativity (cf. Bonotto, 2013; Voica & Singer, 2013; Van Harpen & Presmeg, 2013; Sriraman & Dickman, 2017; Singer, Sheffield, & Leikin, 2017). In a recent handbook chapter, Cai, Hwang, Jiang, and Silber (2015) discuss the progression of problem posing research along ten answered as well as 14 unanswered questions. Amongst others, they ask whether it is feasible to use problem posing as a measure of creativity, pointing at one possible connection between problem posing and creativity. There is, however, still much work to do in this field. Working in both the field of problem posing (Baumanns & Rott, in print) as well as in the field of mathematical creativity (Joklitschke, Rott, & Schindler, 2018), we were intrigued to examine the intersection of both fields (Fig. 1) as indicated by Silver (1994, 1997) or Cai et al. (2015). Ayllón, Gomez, and Ballesta-Claver (2016) conducted a review of this intersection. However, there are some uncertainties (details are explained below) in the content and it is not clear to what extent the review fully reflects the existing research literature. Therefore, this article presents an attempt at a systematic review of studies dealing with both problem posing and creativity published in highly ranked journals.

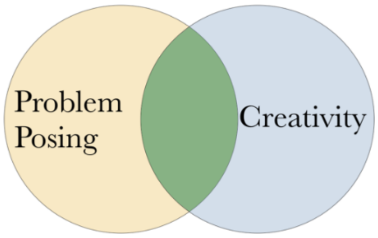

Fig. 1: Intersection of research on problem posing and creativity as focus of this paper

## BACKGROUND

In the following, we provide a current theoretical understanding of mathematical problem posing, mathematical creativity, and their intersection.

## Problem Posing

Problem posing has been emphasized as an important mathematical activity by many mathematicians (e.g., Hadamard, 1945; Cantor, 1966/1932) as well as mathematics educators (e.g., Brown & Walter, 1983; Silver, 1994; English, 1997). As an important companion of problem solving, problem posing can lead to flexible thinking, improve problem-solving skills, and sharpen learners' understanding of mathematical contents (English, 1997). There are two definitions of problem posing, at least one of which is used or referred to in the majority of research papers on the topic. The first definition was proposed by Silver (1994, p. 19), who describes problem posing as the activities of generating new problems and reformulating given problems. Both activities can occur *before*, *during*, or *after* a problem-solving process. The second definition comes from Stoyanova and Ellerton (1996, p. 518), who refer to problem posing as the "process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems". The authors also maintain a categorization for different types of problem-posing situations and differentiate between *free*, *semi-structured*, and *structured* problem-posing situations, depending on their degree of given information.

## Creativity

Solving and posing complex problems often requires creative ideas; particularly in technology and science, this development is very important. Educational research also has an increased interest in research in this field (Singer et al., 2017; Joklitschke, Schindler, & Rott, 2018). Research on creativity goes back to at least the psychologist Guilford (1967) exploring the nature of intelligence. In his work, he differentiated convergent and divergent thinking abilities, the latter encompassing *fluency*, *flexibility*, *originality*, and *elaboration*. These dimensions are apparent in the well-known Torrance Tests of Creative Thinking (TTCT; Torrance, 1974), which is an attempt to make creativity measurable quantitatively. In the field of mathematics education, several researchers draw on this composition to assess mathematical creativity (e.g., Leikin & Lev, 2013; Pitta-Pantazi, 2017). Other researchers (e.g., Liljedahl, 2013) look at creativity using a model consisting of the phases *preparation*, *incubation*, *illumination*, and *verification* and thereby follow Hadamard (1945). In early research, mathematical creativity was attributed exclusively to experts (e.g., Hadamard, 1945) and was therefore an *absolute* characteristic. However, a number of researchers assume that creativity may also be attributed to students, their processes, or products and view creativity as a more *relative* construct (e.g., Leikin & Lev, 2013).

## Intersection of problem posing and creativity

As we explained in the introduction, a considerable part of studies investigating the intersection of problem posing and creativity refers to the articles of Silver (1994 and 1997, resp.), which is why we highlight Silver's key statements in the following.

In 1994, Silver points out that various tests to identify creativity include problem-posing situations; thus, it is reasonable to assume a connection between problem posing and creativity. However, he states that the nature of this connection remains uncertain and needs further investigation. In 1997, Silver considers Torrance's (1974) categories of *fluency*, *flexibility*, and *originality* as key components of creativity and provides instructional suggestions how to foster creative activities in classrooms through problem posing. Furthermore, Silver (1997) emphasizes that "the connection to creativity lies [...] in the interplay between problem posing and problem solving" (ibid., p. 76). In the following, we focus on problem posing and its relation to creativity and describe the findings in this field on the basis of the following research questions: (1) *What kind of (and how many) journal articles exist dealing with the intersection of mathematical problem posing and mathematical creativity?* (2) *To what extent is this intersection conceptualized?*

## METHODS

For this review, we used the preliminary work of two literature reviews on problem posing (Baumanns & Rott, in print) and on mathematical creativity (Joklitschke, Rott, & Schindler, 2018). We focused on (a) databases from the seven A*- and A-ranked journals (Törner & Arzarello, 2012) and on (b) the *Web of Science* within selected categories on mathematics and its education. In the databases from (a), we used the search term *problem posing* for all available years up to 2017. This procedure led to 332 articles. We then read all abstracts and extracted all articles that have *problem posing* either in their titles, abstracts, or keywords; this led to 48 articles. Furthermore, we consulted the database (b) *Web of Science* for the years 1945 to 2017. Excluding the already considered articles from the A*- and A-ranked journals, this led to another 81 articles. Within these 129 articles on problem posing from (a) and (b), we looked for the search term *creativ** within the titles, abstracts, and keywords to identify articles that potentially deal with the intersection of mathematical problem posing and mathematical creativity. In total, only 15 articles (eleven from the A*- and A-ranked journals and four from the Web of Science) remained.

In order to examine those 15 articles in a systematical and criteria-led way, each article was carefully read and assigned to one of the following genres: *theoretical contributions*, *review articles*, *perspective or opinion*, *empirical research with mainly qualitative methods*, and *empirical research with mainly quantitative methods*. Thereafter, the articles were examined with regard to their content. Due to our research question, we concentrated mainly on the conceptualizations of problem posing and creativity and the implementation of the empirical research if there is any. Thereby, the following questions were decisive: *What is the main message of the article? Which theories and conceptualizations are cited? How is the relation between problem posing and creativity represented?* Based on these questions, clusters were formed inductively to classify the articles into coherent groups that represent different approaches at the intersection of problem posing and creativity.

## RESULTS

### Introduction of the reviewed articles (Research question 1)

In the following, the 15 articles are presented and sorted by their genres.

*Theoretical contributions*: Two articles from our data set – the already mentioned articles by Silver (1994 and 1997, resp.) – were considered as theoretical contributions to the intersection of problem posing and creativity. Since the central focuses of the articles have already been covered above, we refer to the background for additional information.

Both articles are widely recognized as milestones in the (back then) young research fields on problem posing, mathematical creativity, and its intersection, respectively.

*Review articles*: One article written by Ayllón et al. (2016) is a review article summarizing central results regarding the relationship between creativity, problem posing, and problem solving. Apart from some inconsistencies and inaccuracies (e.g., wrongly assigned contents), the article clearly fits the topic of our review. However, as the number of articles considered to reflect the state of research of the intersection of problem posing and creativity is limited (three of four cited studies are discussed here as well; the fourth article does not meet our search criteria), the article by Ayllón et al. is not further considered.

*Perspective or commentary*: Two articles were categorized as commentary articles as they neither report on empirical studies, nor provide a theoretical discussion on problem posing or creativity. (1) Haylock (1997) presents examples of tasks designed to identify creativity in 11-12-year-old students. Highlighting *overcoming fixation* as a key component of creativity and referring to Guilford's and Torrance's ideas of *divergent production*, Haylock discusses specially designed problem-solving, problem-posing, and redefinition tasks that have been tested in previous studies. (2) Sriraman and Dickman (2017) discuss the use of *mathematical pathologies* (i.e. unpleasant counterexamples in the sense of Lakatos) to foster creativity in the classroom. In addition to historical examples of pathologies, the authors present examples from current classrooms in which students explore counterexamples or "incorrect" methods leading to correct results (e.g., a misinterpretation of rules to deal with fractions). Sriraman and Dickman then propose using the Lakatosian heuristic (conjecture – proof – refutation), to pose interesting problems that productively deal with pathologies and counterexamples.

*Empirical research with mainly qualitative methods*: In two articles, qualitative empirical studies are presented. (1) Leung (1997) correlates posed problems of 96 grade five students from Taiwan working on 18 different initial situations and describes those creatively posed problems in terms of change of content and context. (2) Voica and Singer (2013) investigate cognitive flexibility (i.e. variety, novelty, and change in framing) in problem posing. They analyze the products of 42 students with above average mathematical abilities working on structured problem posing situations.

*Empirical research with mainly quantitative methods*: In four articles, quantitative empirical studies are reported. (1) Bonotto (2013) investigates the potential of so-called *artifacts* (i.e. real-life objects like restaurant menus, advertisements, or TV guides) to stimulate critical and creative thinking. Additionally, she analyzes problem-posing and problem-solving products from 71 primary school students (stimulated by artifacts), using Guilford's categories of fluency, flexibility, and originality. (2) Singer, Voica, and Pelczer (2017) assess the cognitive flexibility (as an indicator for creativity) of 13 prospective teachers by analyzing the products from geometric, semi-structured problem-posing situations. (3) Van Harpen and Presmeg (2013) investigate the relationship between mathematical problem-posing abilities and mathematical content knowledge among high school students from three different countries. Similar to Bonotto (2013), they analyze the students' problem-posing products using the dimensions of fluency, flexibility, and originality to assess the students' creativity. (4) Van Harpen and Sriraman (2013) also use those dimensions to analyze problem-posing products of 218 high school students from the USA and China.

*Excluded articles*: Four articles (in alphabetical order) had to be excluded for reasons that are outlined below. (1) Ernest (2015) discusses social outcomes of learning mathematics in school by presenting standard aims as well as unintended (and often negative) outcomes (e.g., values, attitudes, and beliefs) and visionary aims (in which he emphasizes mathematical creativity through problem posing and solving) of school mathematics. Ernest uses neither the theoretical literature of problem posing research, nor that of research on creativity; he mentions those terms in his discussion and emphasizes their importance. (2) Patton (2002) uses biographical interviews to trace the recognition of creativity in the lives of famous entrepreneurs and scientists. To interpret his data, Patton uses the *systems theory view of creativity*, which suggests that creativity is not about being unique, but about being the first or being a "problem pioneer". He does cite literature from research on creativity (esp. Csikszentmihalyi), however, he does not use any literature from research on problem posing and, therefore, does not work in field of the intersection of both. (3) In the article by Poulos (2017), the author investigates the way an expert problem poser (a coach of the Greek team for the IMO) poses problems on the Olympiad level by describing two interviews. The author only uses literature from problem-posing research addressing experts' behavior. He does not cite any articles from creativity research and does not investigate the expert's creativity. (4) Singer, Sheffield, and Leikin (2017) wrote the introductory article to a ZDM special issue on creativity and giftedness in mathematics education. Thus, they do not present research results or new theoretical ideas in this article, but rather give a historical overview of research on those topics.

## Cluster formation (Research question 2)

In order to summarize ideas and empirical implementations that can be found in research on a meta level, we will focus on the articles of the categories *empirical research* and *perspective or opinion*. Articles of the categories *theoretical contributions* and *review articles* will be held aside. The remaining eight articles can now be merged inductively into clusters. Some articles are assigned to multiple clusters.

*Cluster I: Problem-posing situations to foster creativity*: The first cluster contains articles that provide examples of problem-posing situations that are especially appropriate to foster creativity. For Cai et al. (2015, p. 17), the question which kind of problem-posing situations are appropriate to promote students' creativity is still unanswered. (1) Haylock (1997) presents tasks for his key components of creativity. For the component of *overcoming fixation*, he designed series of problems in which stereotypical approaches should be discarded. Additionally, he argues that in particular specially designed problem-solving, problem-posing, and redefinition tasks require the component of *divergent production*. (2) Bonotto (2013) investigates the potential of *artifacts* as semi-structured problem-posing situations to identify (see also II.a) and foster critical and creative thinking in the classroom. (3) Sriraman and Dickman (2017) use *mathematical pathologies*, the Lakatosian heuristic, and problem-posing activities to address students' creativity.

*Cluster II: Identifying and investigating creativity through problem posing*

*II.a: Guilford's and Torrance's framework*: This cluster contains all articles that – similar to Leikin and Lev's (2013) analyses of multiple solution tasks – use the categories *fluency*, *flexibility*, and *originality* based on works by Guilford (1967) and Torrance (1974) to investigate the participants' problem-posing products. (1) To measure *fluency*, Bonotto (2013) takes the total as well as the average number of problems created by the pupils working with her artifacts into account. To measure *flexibility*, the posed problems are categorized with regard to the number of details presented as well as the data introduced by the students. To measure *originality*, the rareness of the posed problems is considered: if a problem was posed by less than 10 % of the other pupils, it was considered original. (2) Van Harpen and Sriraman (2013) operationalize *fluency* and *originality* in the same way. *Flexibility* is measured by the total number of categories (e.g. analytical geometry, lengths, area, angles) the posed problems of a student can be assigned to. (3) Van Harpen and Presmeg (2013) use the same operationalizations of all categories as Van Harpen and Sriraman (2013). (4) The article by Leung (1997) could not be classified into this cluster quite as clearly; although Torrance's dimensions are mentioned and the test instrument is also based on the TTCT, these components do not play a role in the empirical part.

*II.b: Other approaches to measure creativity through problem posing*: The third cluster considers the approach by (1) Voica and Singer (2013), and (2) Singer, Voica, and Pelczer (2017). This cluster represents another line of research on mathematical creativity that is based on organizational-theory and discusses creativity in terms of cognitive flexibility (composed as cognitive variety, cognitive novelty, and change in framing) as an indicator for creativity. As a theoretical concept, they refer to the construct of cognitive flexibility to grasp the relationship between problem posing and creativity. As a methodological concept, they especially use the criteria of coherence and consistency of the posed problems. Additionally, Singer, Voica, and Pelczer (2017) consider the two dimensions of Geometric Nature (GN), and Conceptual Dispersion (CD). The GN assesses whether the posed problem is about finding sizes or specific computation (metric), or about geometric reasoning without computation (qualitative). The CD assesses if the posed problems are organized within clearly defined structures or systematically exploiting a configuration (structured), or the posed problems are e.g. disconnected from each other (entropic). They found that cognitive flexibility is inversely correlated to metric GN and structured CD.

## CONCLUSION & OUTLOOK

In 1994, Silver stated that it is reasonable to assume a connection between mathematical problem posing and mathematical creativity; the concrete connection, however, was unknown. 25 years later, the gain in knowledge is still limited. In our literature review, we found only eleven articles in the whole databases of all A*- and A-ranked journals on mathematics education and on the Web of Science that address the intersection of problem posing and creativity. Since our review goes back to the founding dates of journals (e.g., ESM started in 1969, JRME in 1970, and FLM in 1980.), we realized that no articles addressing both problem posing and creativity have been published before 1994.

Analyzing the content of the articles under review, we were able to build coherent clusters focusing on (I) *problem-posing situations to foster creativity* and (II.a) *using problem posing to identify and investigate creativity via Guilford's and Torrance's framework* or (II.b) *other approaches to measure creativity through problem posing*. The inductively built clusters (I) and (II.a) can also be deductively sustained from Silver's theoretical considerations. In 1997, he focuses on fostering creativity through problem posing (and problem solving) which is the key aspect of cluster (I). Additionally, both of Silver's articles (1994, 1997) provide considerations to assess, identify, and investigate mathematical creativity through problem posing by applying Guilford's (1967) and Torrance's (1974) framework as in cluster (II.a). Interestingly, the assumption that problem posing can be used to measure creativity has not thoroughly been investigated, for example by correlation studies.

Apparent limitations of this review lie in the mere description of the articles central methodological and content-related elements. A more in-depth discussion about the chosen theoretical foundations, the research approaches as well as the results cannot be carried out at this point. Furthermore, the aim of this article was to look at the intersection of creativity and problem posing, which is why we did not consider other constructs such as problem solving or giftedness.

In order to expand the dataset of the existing articles, it would be of interest for a future review to also consider the databases of B-Journals (Törner & Arzarello, 2012), seminal collections on problem posing such as *Mathematical Problem Posing* (Singer, Ellerton & Cai, 2015), as well as the papers of the International Group for the Psychology of Mathematics Education (PME) and the International Group for Mathematical Creativity and Giftedness (MCG). Furthermore, additional keywords such as *innovat**, *invent**, and *divergent think** regarding creativity (cf. Joklitschke, Rott, & Schindler, 2018) or *task design*, and *problem formulation* regarding problem posing would extend the range of articles considered in the field of mathematical creativity. This consideration may lead to a wider data base, further clusters, and a better chance of comparing the different research approaches.

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