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EVEN YOUNG CHILDREN ARE ABLE TO GRASP AND APPLY LOGICAL RULES IN MATHEMATICALLY STRUCTURED ENVIRONMENTS. THE PUZZLE OF COGNITION.

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This contribution provides a summarizing insight into the theoretical and empirical research resulting in participation in the project 'Future Strategy Teacher Education – Shape Heterogeneity and Inclusion', funded by the German Federal Ministry of Education and Research and implemented by members of all four faculties of the University of Cologne involved in teacher training. Main foci are types of logical thinking as well as the large range from gifted children to children with special needs.

The Puzzle of Cognition

Experiences causes changes in the brain. How which cognitive procedures take place, is still unknown. The process may end with results such as learning the power of speech, but also with specialized abilities such as solving mathematical problems. There is no doubt that the environment plays a fundamental part in these processes. By using the influence of the environment, it is possible to control the developmental learning processes. The teacher takes care of the external settings conducive to these processes. The fact that learners themselves can generate ideas and exercise their abilities in creative logical thinking is a fundamental objective of modern mathematics education.

Functional Logical Thinking – Predicative Logical Thinking

Basic research on empirical mathematics education has demonstrated, that people differ in having a rather process oriented or object oriented view. Firstly, this was examined by using figural matrices, a task format developed by Charles Spearman (1904) in order to examine fluid intelligence. This resulted in the term pairs functional logical thinking / predicative logical thinking (Schwank 1999). Figure 1 shows a pattern that can be handled in both ways. When using functional logical thinking, figures are seen as being in a process during the rows and columns. For example, in the first row, the lateral curves are first pushed inside, then pulled outside. This principle of construction is also applicable to the other rows and even to the columns, in this case applied to the upper and lower curves. When using predicative logical thinking, the figures are not regarded in their processual connection but concerning their structural similarities: In all rows, the upper and lower curves are invariant, this is also true for all lateral lines in the columns. The resulting figure, a kind of four-leaf clover, is the same for both kinds of logical thinking, only the reasoning demonstrates the underlying difference in the cognitive approach. This is highly significant for the teacher.

Kindergarten and primary mathematics education should take these different approaches into account. If there is no proper reaction to or maybe even perception of difficulties in taking on a functional logical view, difficulties in dealing with numbers and the connected calculation operations are inevitable. Training both views of creative logical thinking poses a remarkable challenge.

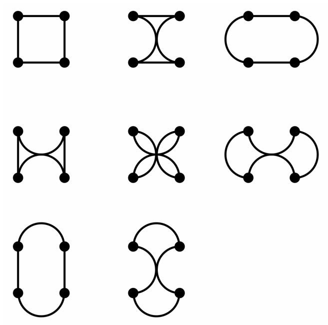

Figure 1: Task for examining functional logical or predicative logical thinking, respectively

Cherishing Children in their Mathematical Talent

One possibility to fathom children's potential of logical thinking is to pay special attention to children that are noticeably mathematically talented during school lessons. On the one hand the performance on itself is important, but on the other hand also what may be deduced by children's behavior.

During a period of 13 years, a total of about 2200 third graders were challenged with mathematical problems (Schwank, 2016). The special trait of these tasks consists of the stimulation of metacognitive activities in the children (e.g. de Jager et al., 2007, Izzati et al., 2018). This means, that they are not only supposed to apply mathematical knowledge but also to think about their activities.

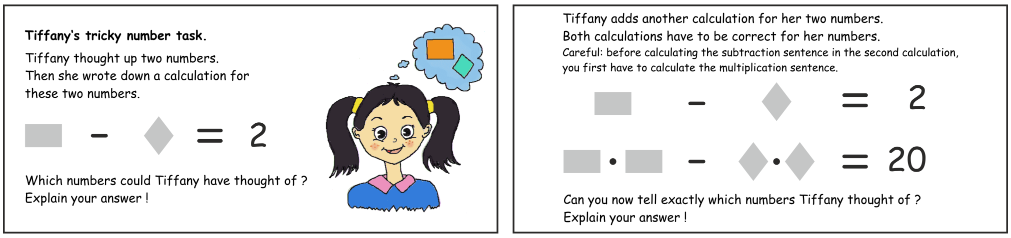

Figure 2: Tiffany's tricky number task

The following passage will discuss the first of the two subtasks in Figure 2. Hereby, the question arises, how and to which extent third graders can independently grasp, that the given equation has to be considered against the background of all natural, i.e. infinite (!), numbers. Below, several examples are presented, for further reading see Schwank & Nowinska (2008).

Type A: Child A1: 6 and 4 / Child A2: 6 and 4 because 6-4 equals two.

Type B:

Child B1: She could have taken the numbers 3-1 or 10-8 or 5-3 or 8-6 or also other numbers.

Child B2: All numbers are possible!!!

Type C

Child C1: 6 4, 7 5, 8 6, 9 7, 4 2, 3 1, 2 0. One can often reach 2 and that is why there are several possibilities how to reach 2.

Child C2: Tiffany could: 3-1=2, 4-2=2 and doing until ∞ - nearly ∞ = 2. Tiffany couldn't: 3-2=2 or 5-100=2 and 5+0=2.

Child C3: 8-6=2 10-8=2 Because I can subtract from any number so much that I get 2.

Type D

Child D1: Tiffany could take 7-5. The number 2 always had to fit in between.

Child D2: 3 and 1, 5 and 3, 6 and 4, 7 and 5, 8 and 6, 9 and seven. All numbers where one number has 2 more than the other.

Child D3: All, as long as one of them is two numbers higher.

Child D4: 4 a. 2, 6 a. 4, 8 a. 6, … The numbers have to be 2 numbers apart. The higher number has to be at the beginning.

Child D5: The first number has to be greater by 2 and there are lots of possibilities.

Type E: Child E1: (Crossed out: Always two numbers that are two apart, the higher one first.) The numbers always have to be apart as much as the result is.

Just as with type A, there are children that are content with just a single example or, as seen with type B, take several, even up to all numbers into account, but do not consider the specific number traits necessary to solve the equation. Concerning type C-E, a processual view – along with a functional logical approach – is taken, to handle the multiple possibilities as well as further aspects. In the case of child C1 there are – along with an explanation – ‚several possibilities‘. Child C2 starts a process leading to the infinite (and names counter examples). Child C3 catches the infinite amount of numbers with ‚any number‘, the number to which something happens (without taking the special cases 1 and 0 into consideration). Type D focusses on the constant distance of the two inserted numbers. Child D1 thereby chooses the expression ‚always‘, Child D2 ‚all numbers‘ (with a certain characteristic) and Child D3 simply ‚all‘ (the context thereby revealing that all numbers are meant). Child C4 also mentions that the larger of the two numbers has to be the first one and Child D5 points out that, moreover, many more possibilities can be found. Child E1 is remarkable in a special manner. While firstly it concentrates on the distance between the two selectable numbers and puts the larger one first, it "corrects" itself and thinks beyond the task. It generalizes the task from *number a - number b = 2* to *number a - number b = number c*: The two selectable numbers always have to be apart as much as the result. Creative logical mathematical thinking requires that one takes the time to dive into the mathematical content concerning the given issue. Carefully looking around and experiencing the content is helpful for the generation of individual ideas.

Zero is an Even Number

Creative logical thinking starts at kindergarten age (Schwank & Schwank 2015). There mathematical playworlds are the key to building a solid foundation, they enable the children to generate ideas without having to use mathematical terminology or notation. The empirical basis using mathematical playworlds to foster the early development of mathematical thinking is a multi-year field study in four kindergartens with more than 300 children. A first impression is given by the following example. The game leader starts by placing a blue cellular rubber disk in a mathematical playworld called spiral stairs of calculation (Schwank & Schwank, 2015) as demonstrated on the left in Figure 3 and asks the children "What catches your eye?". As one child notices that there are the same number of balls above and beneath the cellular rubber disk, the game leader continues with the question "Where else might this be possible?". Afterwards, the children place further cellular rubber disks as displayed in the middle of Figure 3. Even before the game leader can once again step into action, the children suggest that the space with zero balls also requires a cellular rubber disk in order to complete the rule that is to be followed. The game leader thereupon asks the children "Why so?"

Child 1: Because, uhm, then there is nothing on either side.

Child 2: Uhm… because, because, because with zero, there, there, there, there isn't any, nothing on each half. Hence, this is equally many.

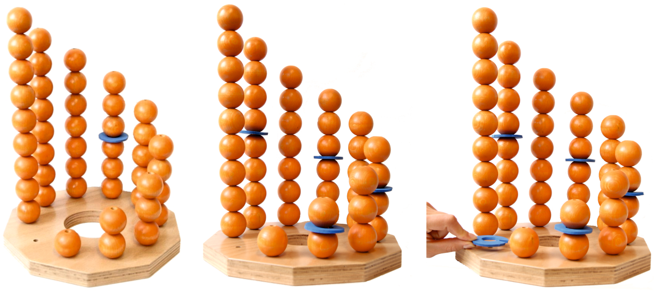

Figure 6: Development of the idea, that the number zero also has the feature of being divisible in two numbers of equal size (and therefore in mathematical terminology being an even number)

Even young children are able to grasp and apply logical rules in mathematically structured environments. The use of 'specialist' terms (e.g. even, odd numbers) is an issue of later development.

Future Strategy Teacher Education – Shape Heterogeneity and Inclusion

In the ongoing government-funded project 'Future Strategy Teacher Education' (German Federal Ministry of Education and Research, 2019) the research and development are continued and extended with regard to children with special needs. We will be presenting first results of a study on six hearing impaired children (8-10) as well as results of a case study on two physically handicapped children (10-11). The leading question is how to promote functional logical thinking given that previous studies displayed this being the basis for the understanding of first important arithmetical ideas such as numbers and their dynamic relationships or the mechanism of the decimal numeral system (Schwank & Schwank, 2015).

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