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ANALYSIS OF EYE-MOVEMENTS DURING FUNCTIONAL VERSUS PREDICATIVE PROBLEM SOLVING

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Abstract: *The theory of functional / predicative thinking, once developed to describe differences in students' behaviour while solving programming tasks, is applied to analyse eye-movements while solving tasks of visual pattern completion (QuaDiPF-tasks, Schwank 1998/2000). Predicative thinking requires that in order to meaningfully complete the pattern the subject has to get involved with the logic of the static structure of the pattern, functional thinking requires to get involved in a dynamic reading of the logic of the pattern. The QuaDiPF-tasks proved to be useful in other experiments to predict typical functional or predicative behaviour of the subjects. The eye-movement-study is a second approach after an EEG-study to use not only qualitative methods for the classification of problem solving behaviour, but also quantitative ones.*

1. Qualitative investigations of functional/predicative thinking

In the very first days of the theory of predicative versus functional thinking (Schwank 1986, 1993-1995) we used the nonverbal intelligence test APM (Advanced Progressive Matrices) from Raven (1965) to balance our different groups of subjects spread over such different countries as Germany, Indonesia and China (Marpaung 1986, Xu 1994). The focus of interest was on differences in students' behaviour while solving some programming problems using different types of microworlds. The label "*predicative*" was used to characterize a problem solving behaviour highly orientated at and sensible for features, relations and judgements, whereas the label "*functional*" was used to characterize a problem solving behaviour highly orientated at and sensible for courses, modes of actions and effects. The experiments were run in the form of constructive teaching experiments (Cobb et al. 1983), hence, qualitative analysis methods were to apply. Although one could have expected strong cultural influences on the cognitive processes involved when solving our programming tasks, the results show, that knowledge about the national relationship cannot help to describe or to predict the individual types of problem solving behaviour of our subjects. Instead, the distinction of functional and predicative thinking was very helpful. The results show, that students tend to have a strong preference for either of the two thinking styles, which shows up most clearly in cases when students are quite intelligent and do reach the limits of their intellectual power when challenged by specific tasks.

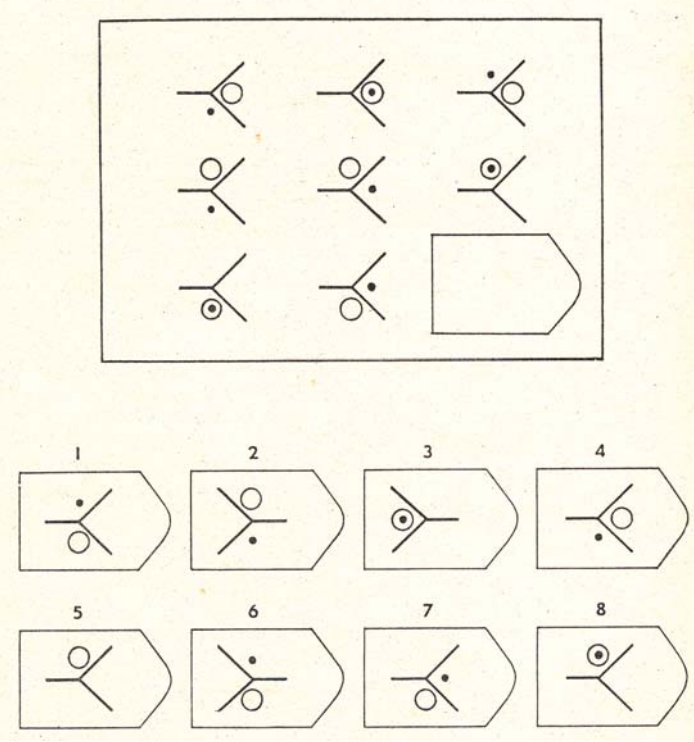

Fig. 1: Typical Raven-task (APM II-14)

Only later we did begin wondering why students, who were remarkable for their good functional behaviour in our programming tasks, had the same good results in the APM-test as students, who were remarkable for their good predicative behaviour. At first glance, it seems that solving APM-tasks requires getting involved with the logic of the static structure of the pattern and therefore one has to recognize the regular recurrence of certain features of the elements (e.g. shape: square, circle …; size: small, big, …, position: left, right, …). One of the possible predicative analyses of the APM II task No. 14 (Fig. 1) goes as follows: Each figure consists of three objects: an upsilon, a point and a circle. The upsilon is the same in each figure. In each row the circle is at the same place. In each column the point is at the same place.

The interesting point is, that there does also exist a functional strategy to solve this task: Perceiving the pattern a process is invented which produces the last element in a row or column. In each row the point moves around, and in each column the circle moves around. The object around which the movement goes on remains stable.

The moment we realized these two different approaches of inductive thinking (Schwank 1996) while solving matrices tasks we had the idea for a new tool to test subjects how far they are able to think in a predicative way and how far they are able to think in a functional way.

We invented new tasks, which differ considerably in their level of pure functional and pure predicative difficulty (QuaDiPF-Test; Schwank 1998/2000). As we are interested in the decomposition of thinking processes, we eliminated the possibility to start visually matching procedures by not offering a sample choice of 8 solutions as in the original APM-test. Instead of such a kind of choice the subjects are asked to draw their solution figure and to argue why their figure fits the pattern. The sessions are videotaped and the reasons given by the subjects are qualitatively analysed.

In several experiments we use QuaDiPF-tasks as well as other tasks like term rewriting tasks (Cohors-Fresenborg 2001; Striethorst, in press) or programming tasks (Xu 1994; Armbrust, in press). The results of the QuaDiPF-test allow a good prediction whether a strong functional or a strong predicative behaviour is to be expected from the subjects in cases of the other problem solving tasks.

2. Quantitative investigations of functional/predicative thinking

EEG-Studies

Compared with the complex tasks used in our experiments about programming behaviour or term rewriting the QuaDiPF-tasks are very simple - actually in a very specific sense. The only visible activity the subject has to carry out while thinking about a QuaDiPF-task is to move his / her eyes. Through that and because of the simplicity of presenting these tasks for mental exercises suddenly EEG-experiments on functional and predicative thinking became possible. So far we have been running two of such experiments together with Jan Born and members of his research group at the Medical University of Lübeck, the first of them has been completely evaluated (Mölle et al. 2000, Schwank 1999). In this study the EEG was recorded in 22 young men while solving QuaDiPF-tasks. The results are that the EEG complexity during predicative thinking decreased in comparison to functional thinking and mental relaxation, with this reduction being most pronounced over the right and paretial cortex; a reduction in dimensional complexity during functional thinking as compared to mental relaxation which was concentrated over the left central cortex, although significant, was less clear.

Eye-Movements-Studies

Having known for long about the possibilities of eye-movement-studies (Galley 2001), we were looking for some partners who would like to co-operate with us. Carpenter et al. (1990) made an attempt to analyse the manner how subjects solve the Raven Matrices using eye-tracking methods. In summary they stated: "*The processes that distinguish among individuals are primarily the ability to induce abstract relations ...* " (p. 404) and the only pure functional Raven task they excluded from analysis: "*Problem was not classifiable by our taxanomy*" (p. 431). The question is of course, what about functional problem solvers in comparison to the predicative ones. Finally we met Franz Mechsner, Max Planck Institute for Psychological Research (MPIPR), Munich, who agreed to run a joint experiment in one of their laboratories and we started with about 20 subjects in summer 1999. The screen resolution in this first pilot study was not the best (640x480). As partners in the new course of study "Cognitive Science" at the University of Osnabrück we got the possibility to use the new equipment for eye-movement studies, e.g. now we can work with a screen resolution of 1024x768. In a second pilot study we tested 12 subjects in Osnabrück using an EyeLink system from SensoMotoric Instruments (for technical details see http://www.smi.de). In the sessions which are videotaped, the subjects get the tasks presented on a screen, they wear a headband (Fig. 2) with two ultra-miniature high-speed cameras to get there eye-movements-data (saccades, fixations). After having solved a task they are asked to draw their solution figure and to argue why their figure fits the pattern well.

Fig. 2: Student wearing an EyeLink headband solving QuaDiPF-tasks

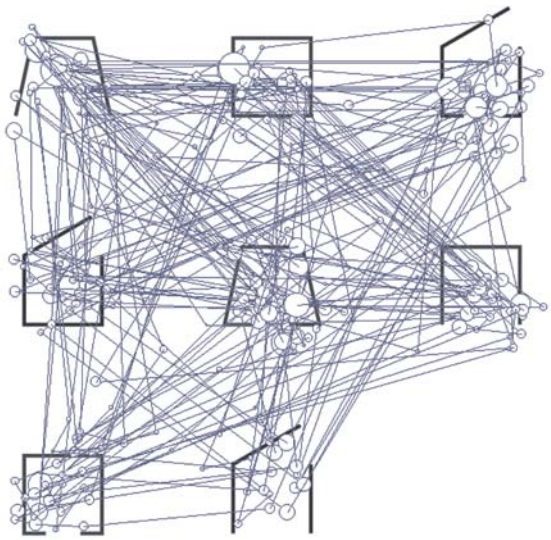

Fig. 3a: Entire eye-movements solving the task
Duration: 01:11,344

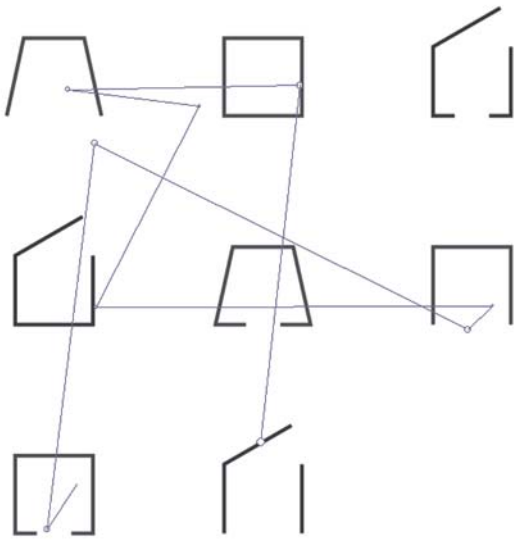

Fig. 3b: Start orientation
Time: 00:02,310 / Duration: 00:01,155

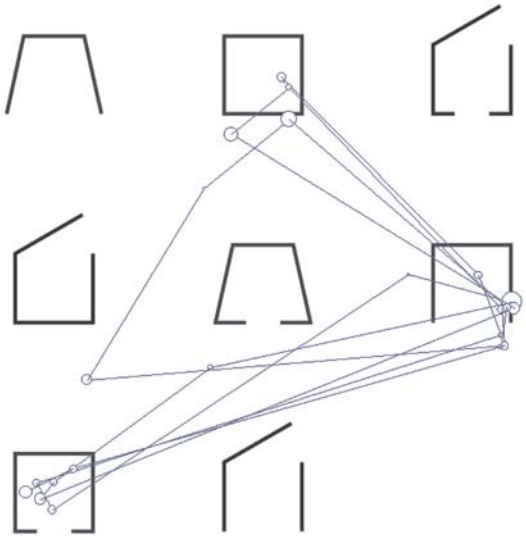

Fig. 3c: Realising the set of "squares"
Time: 00:22,642 / Duration 00:05,166

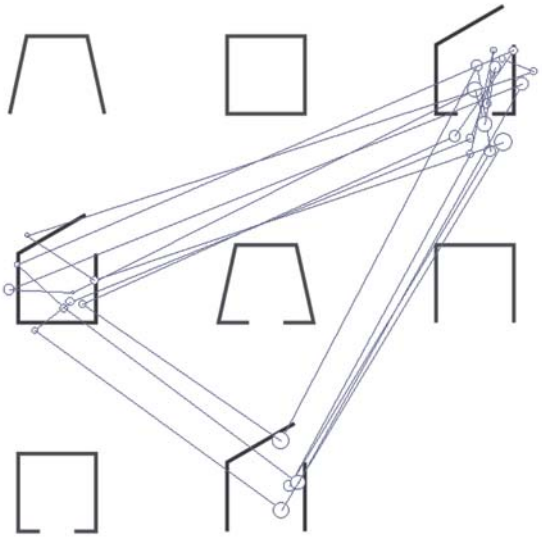

Fig. 3d: Realising the set of "houses"
Time: 00:28,541 / Duration:00:07,445

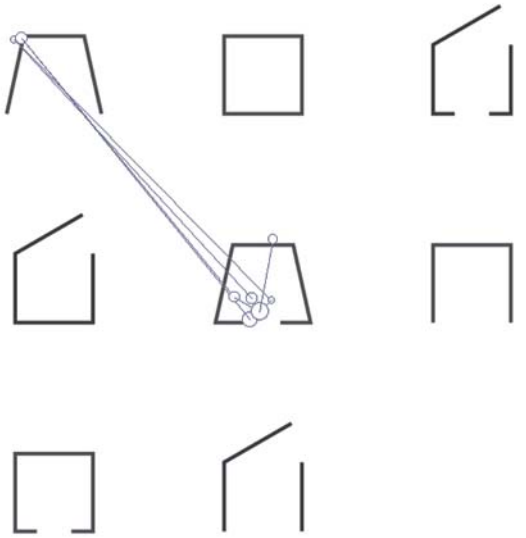

Fig. 3e: Realising the set of "trapezia"
Time: 01:02,880 / Duration: 00:02,336

Figure 3a shows the eye-movements of a predicative solution. The data is scanned regarding interesting sequences of eye-movements: which parts of the pattern are looked at in which sequences. Fig. 3b-3e show such sequences. Time and duration are given in [min]:[sec], [msec]. The self-explaining predicative argumentation of the subject is given below. [E1-E8 refer to the single elements of the figures reading from left to right, starting in the first row.]

Predicative Problem Solving - Example

S[ubject]: (*Finished the drawing*) So. Well, it's not very nice in shape. It is supposed to look like (*S points to E[lement]1*) this one closed. Hm, well, I'll give the following reasons for that, hm, there are three different shapes each time. (*S points to the elements in the first row*). Once it has (*S points to E1*) - they have no bottom line each time. Once with (*S points to E2*) a half full, hm, empty, yes, half a bottom line and once with none at all. This can be seen in there. Once full (*S points to E2*), nothing at all (*S points to E6*) and once half (*S points to E7*). Once here (*S points to E3*) hm half, once full (*S point to E4*) and once none at all (*S points to E8*). And here this is missing (*S points to E1*), then, once none at all (*S points to E5*), once half and then it has got (*S points to E9*) to be in full. I would say so.

V[ersuchs]P[erson]: So. Ja, also formschön ist es nicht. Das soll so aussehen, wie (*VP zeigt auf E1*) das hier zugeschlossen. Ähm, das begründe ich jetzt so, ähm, hier sind jeweils drei verschiedene Formen. (*VP zeigt die Elemente der ersten Zeile*) Einmal hat das (*VP zeigt auf E1*) haben die jeweils einen leeren Boden. Einmal einen (*VP zeigt auf E2*) halb voll äh leeren ja einen halben Boden und einmal überhaupt keinen. Das kann man hier dran sehen. Einmal voll (*VP zeigt auf E2*), gar nichts (*VP zeigt auf E6*) und einmal halb (*VP zeigt auf E7*). Einmal hier (*VP zeigt auf E3*) äh halb, einmal voll (*VP zeigt auf E4*) und einmal gar nicht (*VP zeigt auf E8*). Und hier fehlt das (*VP zeigt auf E1*) dann einmal gar nicht (*VP zeigt auf E5*), einmal halb und dann muss (*VP zeigt auf E9*) einmal voll sein. Würd' ich mal sagen.

Functional Problem Solving - Example

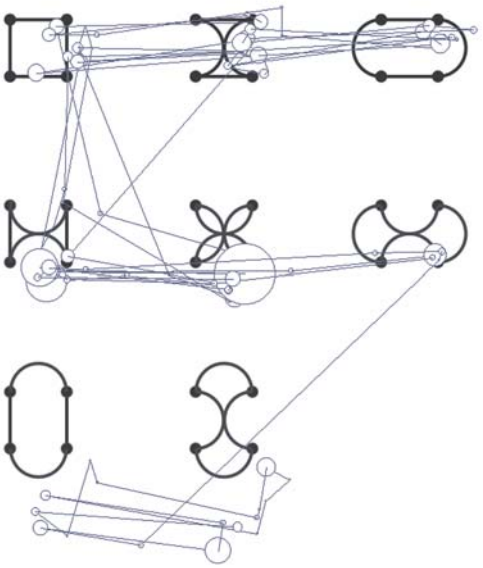

Fig. 4: Entire eye-movements
Duration 00:23,236
The callibration at the bottom of the screen wasn't that succusfull.

S[ubject]: Hm. Left and right, so they move inside as a circle and then outside.
E[xperimenter]: Hm.
S: This repeats every time. (*S points to the left and right side of the "square" in E1 and then goes over the curves in E2 and E3*).
E: Is there also an explanation for the columns? This was an explanation for the rows.
S: Here? Or what? (*S goes over the columns, one after the other*)
E: Yes.
S: I did not consider them. Yes, this might be ... one moment. - Yes, it's the same, basically. First to the inside, then to the outside.

V[ersuchs]P[erson]: Äh. Links und rechts, die gehen also als Kreis nach innen und dann nach aussen. (V[ersuchs]L[eiter]: Ähm.) VP: Jedesmal wiederholt sich das. (*VP zeigt auf die linke und rechte Seite des "Quadrates" in E1 und fährt dann die Bögen in E2 und E entlang*) VL: Gibt's noch ne Erklärung in den Spalten? Das war ja in den Zeilen erklärt. VP: Hier? Oder was? (*VP durchfährt nacheinander die Spalten*) VL: Ja. VP: Da habe ich nicht drauf geachtet. Ja ist vielleicht das ... en Moment. - Ja ist das selbe praktisch, ne. Erst nach innen, dann nach aussen.

This functional solution - the figures result from their predecessors by moving - is interesting for several reasons. After short reflection a fairly well-founded solution is given. Thereby the subject used only information he checked with his eyes in the rows as it is obvious from his eye-movements-data (Fig. 4). The experimenter's demand for further explanation shows the subject being aware of that.

In case of our EEG-studies we had to decide that the subjects give the reasons for their solutions only after they created them mentally, because any physical movement would have made the EEG-data unsuitable for further analyses. It was never clear of which status the explanations, given in retrospect, were. What is their relationship to the thoughts of the subjects at the moment they were developing their solutions? It might be, that the verbal explanations are more or less nothing but nice sentences which the experimenter would like to hear, and thus produced for this purpose. E.g. in a particularly unfavourable case, the subject could have thought functionally, but argued predicatively. Even though it is fascinating to investigate brain mechanisms more directly via EEG-methods it does increase insight to investigate eye-movements as seen here.

The eyes' scanning of the QuaDiPF-tasks, controlled by the brain, at least shows the process of attempts to orientate oneself in the task, where the gaze gets caught, which parts of the task - in which sequences - are preferred compared to those which are disregarded.

3. Outlook

So far, we know, that we find traces of eye-movements, which fit perfectly to the argumentations of the subjects. It seems difficult (or even impossible) to cheat with the eyes, when using the eyes as an essential tool during thinking processes. When we were learning to use the technical equipment for the investigation of eye-movements several members of our research group checked the usage of the EyeLink headband and tried to simulate the solving of known QuaDiPF-tasks. The reproduction of a solution or the attempt to simulate predicative/functional thinking (which requires guarding the thoughts) is accompanied by quite different eye-movements than those during actual thinking processes creating a new solution without specific constraints.

Dependent on the type of a QuaDiPF-task the analyses of the eye-movements show whether a solution was produced in a predicative or (probably) in a functional way. In one type of predicative QuaDiPF-tasks it is even possible to distinguish between two possibilities of predicative problem solving. In Fig. 3 the triangles along which the eyes move are typical for the predicative approach "creating sets taking into account typical features". Another predicative approach is to break down the elements in their components (bottom, walls, top) and to check the regularities of their occurrences in the rows and columns: in each row there are once an open bottom, once a half-open bottom, once a closed bottom; further there are twice straight walls, once bevelled walls; finally there are twice closed tops, once a sloping top; the same goes with the columns. The eye-movements from this kind of predicative solution don't proceed along such triangles as presented in Fig. 3, instead they proceed along the rows and the columns.

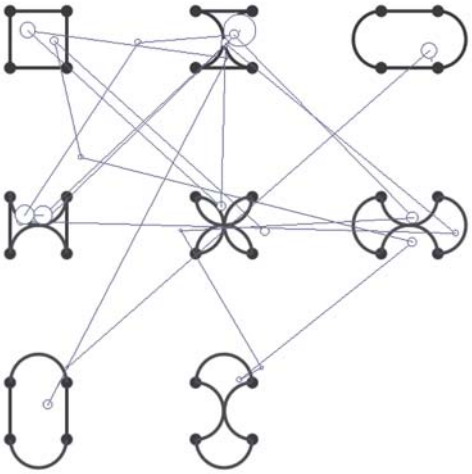

Fig. 5: Searching for pairs
Time: 00:08,953 / Duration: 00:06,355

Eye-movements like the one given in Fig. 4 confront us with problems. We find similar eye-movements in cases of the following suitable predicative procedure: Analogous to the way described above the elements are broken down in their components (bottom, walls, top), then it is very easy to see, that in each row the bottoms and the tops remain the same, the same goes with the walls in the columns. Unfortunately, we are only sure, that it is predicative behaviour we are dealing with, if a subject builds sets, which can nicely be seen in the eye-movements (Fig.5): solving the task given a subject could argue, that there are always pairs of elements, the "square" is missing his partner, the figure in the middle is just the center point. In fact, so far, the first analyses of about 30 subjects show, that the set-building is one of the most striking characteristics in the eye-movements of predicative behaving subjects. In case of the QuaDiPF-task given in Fig. 5 the situation is not satisfying; because the predicative set-building approach is of lower quality than the predicative decomposition approach. Hence, such tasks are not suitable for distinguishing quite good predicative problem solvers from quite good functional problem solvers exclusively by means of the analyses of eye-movements.

In further experiments we will use a further type of functional QuaDiPF-tasks (Fig. 6), for which in the predicative approach only a set-building approach helps, but not a decomposition approach. In addition we will pay more attention to such QuaDiPF-tasks for which the degree of difficulty differs depending on whether one tries to find a predicative or a functional solution. E.g. the QuaDiPF-task given in Fig. 7 is rather difficult, but it is easier to find a solution based on functional arguments than to find a similarly good solution based on predicative arguments. Again, here, the set-building approach is only second class. We expect, that - as in the past - only in rare cases we will find predicative problem solvers, who read the rows and lines and thereby elaborate a good predicative solution. These subjects will stand out from the others because of the short times they need to solve pure, difficult predicative QuaDiPF-tasks like the one given in Fig. 3. And, of course, we will develop more QuaDiPF-tasks, which fulfil the conditions of the meaningful measurement of eye-movements.

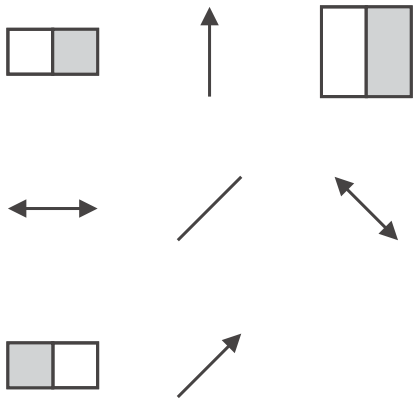

Fig. 6: QuaDiPF Set C - 17

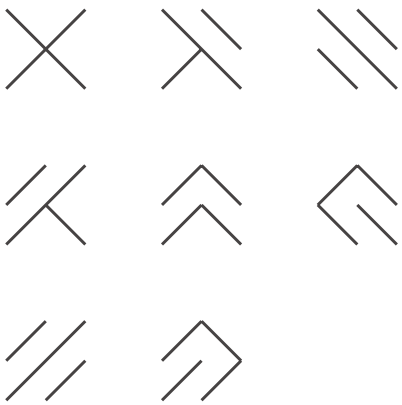

Fig. 7: QuaDiPF Set C - 13

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