ON PREDICATIVE VERSUS FUNCTIONAL COGNITIVE STRUCTURES

Inge Schwank
Institute for Cognitive Mathematics, Department of Mathematics and Computer Science,
University of Osnabrueck, D-49069 Osnabrueck
schwank@mathematik.uni-osnabrueck.de

Abstract: *Predicative thinking is thinking in terms of relations and judgments; functional thinking is thinking in terms of available actions and achievable effects. Depending on the way of thinking the orientation in the world, the type of sources for getting insight are not the same. E.g. it should be visible in different eye movements. In addition to our qualitative experiments, recently we started to run a study based on EEG-methods while students were solving logical pattern fitting tasks. The EEG complexity during predicative thinking decreased in comparison to functional thinking and mental relaxation, with this reduction being most pronounced over the parietal and right cortex. A reduction in dimensional complexity during functional thinking which was concentrated over the left central cortex, although significant, was less clear.*
Keywords*: cognition, EEG, eye movement.*

1. Introduction

Concerning cognitive models as sources for intelligent behaviour like planning or learning different approaches have been studied during the years. There is broad agreement - since the time of Plato - that language and - as a newer approach in our century - visualization is a tool of thinking. Together with the cognitive turn as movement against behaviourism lots of statements indicating that thinking equals language were set.

There are good reasons to accept another cognitive base of elaborated thinking, which is related in a specific sense to motoric-guided cognitive representations. At an interdisciplinary conference on “Thinking and Speaking” the mathematician van der Waerden (1954) pointed out, that thinking in motoric terms is on top level in creative mathematical work: it might be that someone is a visualizer and therefore has more benefit in thinking in images, but for sure the words which are used to label the mathematical concepts only play a minor role. This seems to be in contrast e.g. to what is used of the Piagetian theory of levels of cognitive development, where motoric representations are on the bottom of the hierarchy and it is the formal representations which are on the top. In our theory we use the term *functional thinking* for motoric thinking in the sense of van der Waerden (which includes only such motoric actions useful for productions) and we contrast it with a kind of thinking that doesn't care so much on dynamics but on static structures and the embedded complex relationships, this latter we call *predicative thinking*. Up to now there is not much clearness about the relationship of functional/predicative thinking and all kinds of visualization and imagery. It seems to be clear that in case of predicative thinking good language knowledge is useful, because it is of much advantage in creating an adequate structure in a problem solving situation to use word-labels to build up this specific structure.

Beyond the fact that the functional way of thinking plays an important role in mathematical thinking its range of appliance is even much broader. Bateson (1980[4], 120-121) presented a nice example concerning the difference we are interested in:

»We do not notice that the concept "switch" is of a quite different order from the concepts "stone," "table," and the like. Closer examination shows, that the switch, considered as a part of an electric circuit, *does not exist* when it is in the on position. From the point of view of the circuit, it is not different from the conducting wire which leads to it and the wire which leads away from it. It is merely "more conductor." Conversely, but similarly, when the switch is off, it does not exist from the point of view of the circuit. It is nothing, a gap between two conductors which, themselves exist only as conductors when the switch is on. In other words, the switch is *not* except at the moment of its change of setting, and the concept "switch" has thus a special relation to time. It is related to the notion "change" rather to the notion "object".«

In some studies (e.g. Schwank 1993a, Schwank 1994) on cognitive ways of learning basic concepts of computer science we used within a set of bricks (Dynamic Mazes, www.ikm.uos.de/aktivitaeten/dynamische_labyrinthe.htm) a mechanical switch (Fig. 1) for representing a tool for case distinction. For instance, by means of this switch organisational problems like automatical bottle-selling procedures can be solved. The key point is to plan actions and thereby to anticipate their influence on later occasions, which is a typical functional requirement. Predicted difficulties of part of the students based on their individual problems dealing with functional concepts arose (e.g. Schwank 1994).

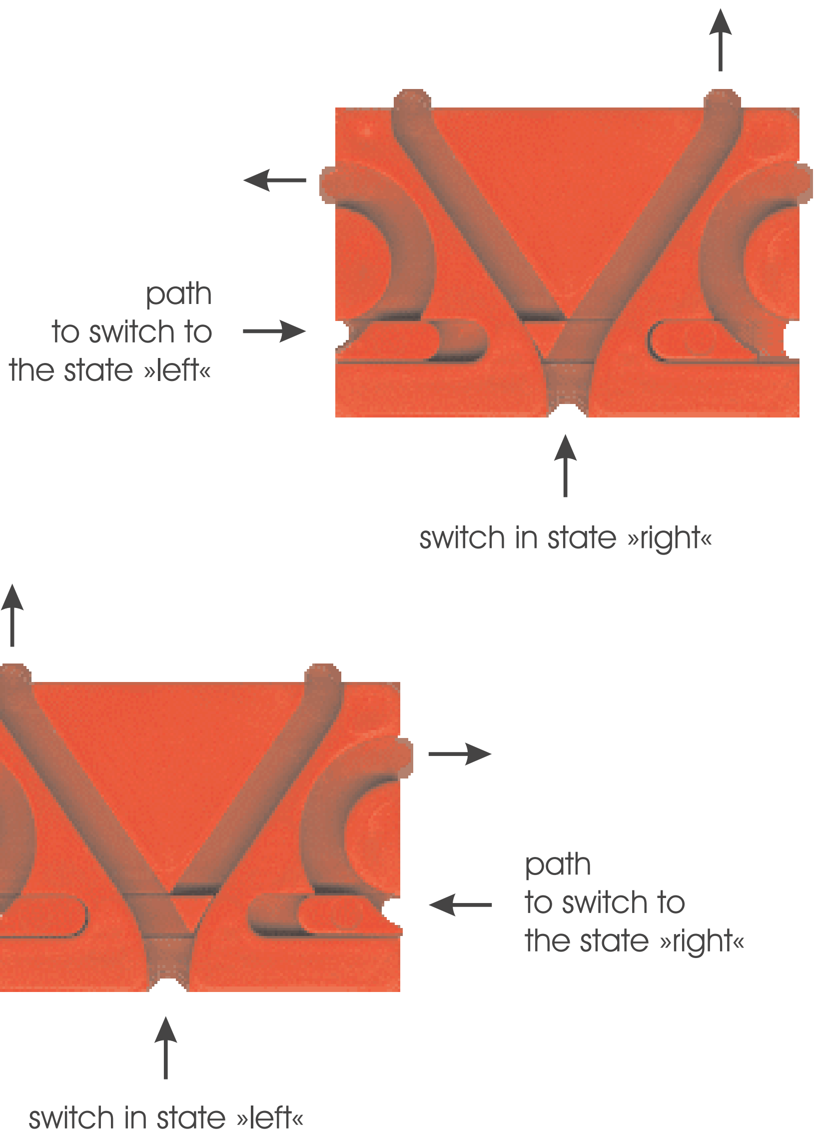

Fig. 1: Switch of the Dynamic Mazes
(Test it.)

In the following we first give a short overview of our theory and than present some examples of short logical tasks, which we are using at present to check abilities and preferences of subjects in using a functional or a predicative cognitive structure.

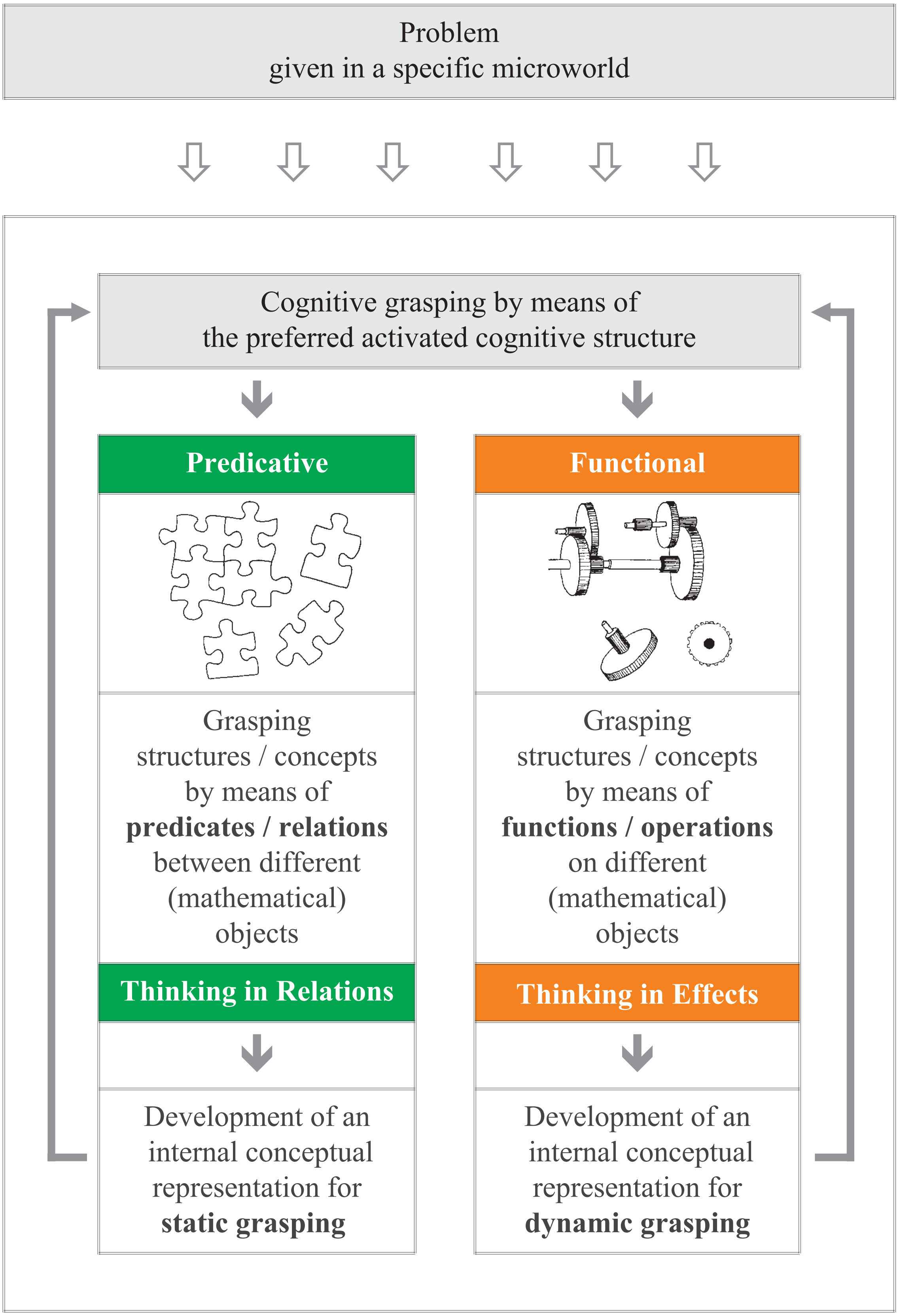

Fig. 2: Predicative versus functional cognitive organisation (cf. Schwank 1995)

2. Predicative versus Functional Cognitive Structures

We distinguish between static and dynamic mental modelling as a characteristic of the individual cognitive structure in terms of predicative versus functional thinking (Fig. 2). Predicative thinking emphasizes the preference of thinking in terms of relations and judgments; functional thinking emphasizes the preference of thinking in terms of courses/effects and modes of action (cf. Schwank 1993a, 1996). For an overview of the experimental testing of the theory see Schwank (1995, 108-115). Research has also shown that it is quite rare to find female subjects who behave in a functional way (see also Schwank, 1994) or that functional and predicative thinking occurs as well in Indonesia (Marpaung 1986) and China (Xu 1994).

The given diagram (Fig. 2) has to be read spirally in chronological order. The black arrows describe circles in order to consider that the internal tools of the conceptual representation influence that which will be grasped cognitively. In consequence the further development of the internal conceptual representations is interfered. The observed differences in behaviour partly are explained in such a way that both kinds of cognitive structures are not applied equally which results in a different development of a more static or a more dynamic internal conceptual representation.

The category of individual cognitive structures has to be separated from the category of individual cognitive strategies. We distinguish between a conceptual, top-down organising, and a sequential, more interactive approach (Cohors-Fresenborg & Schwank, 1996). Predicative / functional refers to the tools of thinking, conceptual / sequential refers to the global organisation of the problem-solving process. Concerning links to other cognitive theories such as declarative/procedural see e.g. Schwank (1993b).

3. Examples

For a predicatively structured person the central point of his or her analysis concerning a complex situation is to break it down into different conceptual pieces and to invent a logical structure which describes the network of the relations between these pieces. For a functionally structured person the central point is to arrange the going through the production as a complex process in which different strengths control, determine or promote each other. For the former the mental model describes the logical structure, for the latter it describes the organisation of work flow in time.

To show the benefit of our cognitive theory for e. g. a cognitive approach in business reengineering (Cohors-Fresenborg & Schwank 1997) or a cognitive science approach in computer programming (Schwank 1993 a,b) we have designed different studies which are run with single subjects using different settings: fitting figures in matrices (QuaDiPF), organising processes in a microworld (OPM). QuaDiPF (Schwank, 1998) is a qualitative diagnostic-instrument to determine the preferred cognitive structure, predicative versus functional. In OPM those tested have to solve a sequence of organisational problems with the specific microworld Dynamic Mazes (cf. Cohors-Fresenborg, 1978, www.ikm.uos.de/aktivitaeten/dynamische_labyrinthe.htm). This is the mechanical realisation of a mathematical idea of automata which is equivalent to the Turingmachine. We know from our studies that this setting in the beginning supports the functionally structured subjects. While solving the more complex problems a predicative cognitive structure is more successful. Here we concentrate on QuaDiPF because the setting is much simpler and not as time consuming as OPM. Finally this newer analysing tool is even usable in EEG-measurement-environments.

3.1 Fitting Figures in Matrices: QuaDiPF

We use tasks such as those in common intelligence tests (e. g. Raven, 1965) to find a missing figure, which fits suitably into a set of 8 given figures arranged in a matrix. In a clinical interview each subject has to invent and draw the missing figure in the matrix (instead of selecting it from a given set as usual). The subject has to argue why he or she drew this very figure. The analysis of the videotapes shows that a predicative and a functional way of mentally modelling the task exist. In a predicative mental model the subject uses *predicative tools*, e.g. looking for properties, inventing general laws. So, in the given example (Fig. 3a) the subject tries to structure the image. Each figure consists of three objects: a star, a point and a circle. The triangle is the same in each figure. In each row the point is at the same place. In each column the circle is at the same place. In

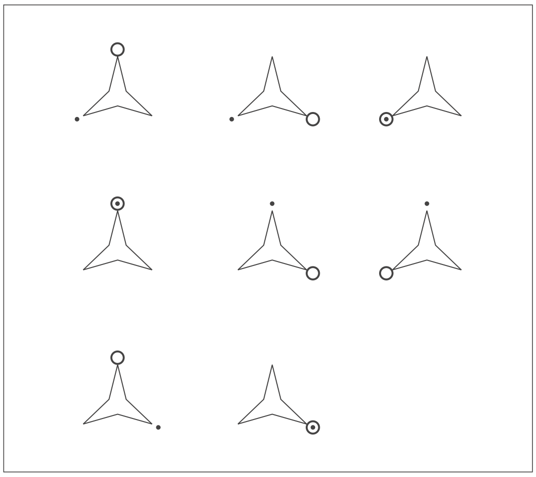

Fig. 3a: QaDiPF-Example (Schwank, 1998)

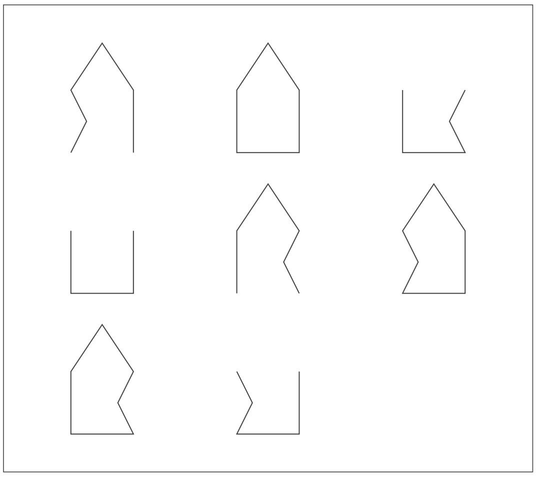

Fig. 3b: QaDiPF-Example (Schwank, 1998)

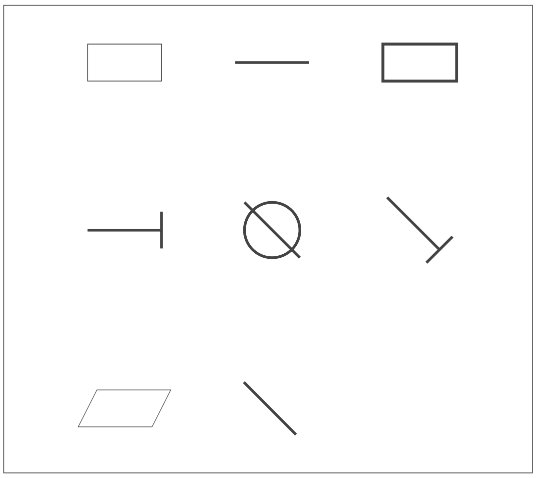

Fig. 3c: QaDiPF-Example (Schwank, 1998)

a functional mental model the subject uses *functional tools*, e.g. invents a process which produces the last element in a row or column. In each row the circle moves around, and in each column the point moves around. The object around which the movement takes place does not change. In both ways of dealing with the problem the result is the identical.

Besides tasks such as 3a we also invented tasks which are either easier using a predicative analysis or a functional one, so that in the end we established a new type of intelligence test. Fig. 3b shows an example in which a predicative analysis is useful to construct a working mental model. The main idea is to invent a structure by arranging the properties. One could, for example, proceed as follows: three types of figures exist (closed figures, figures which are open at the top and figures which are open at the bottom) which each have straight walls, bent left walls and bent right walls. The figure with an open bottom and straight walls is missing (composition of predicates).

Fig. 3c shows an example in which functional analysis is useful. The main idea is that the figures in the middle row and the middle column symbolise operators. One could, for example, proceed as follows: in the first row the first figure is given thick lines by means of the operator. In the first column the first figure is pushed by the operator and transformed into a parallelogram. In the second row the first figure has to be turned by means of the operator. In the last line as a consequence the first figure has to be turned and it has to be given thick lines (concatenation of operators).

We have designed the tasks in QuaDiPF in the form that the subjects have to explicitly construct the missing figure instead of selecting it from a given set of possibilities, for the following reasons: we are interested in the nature of thinking processes and the omission of possible solutions makes the tasks more difficult. Furthermore, we are interested in the individuality of problem-solving: a given set of possible solutions could influence the way in which the tasks are analysed. As a consequence our methodology is rather a qualitative one than a quantitative one.

In the literature it is discussed that solving this kind of tasks requires especially inductive thinking (e. g. Klauer 1996). Our findings show that not only one kind of inductive thinking exists. In a predicative model induction means abstraction. The result is a predicate which is fulfilled by the given examples. In a functional model induction means generalisation. The result is a function which produces the given examples (cf. Cohors-Fresenborg & Schwank, 1996).

4. EEG-Study

Fig. 4: Phase in the EEG-Study

Together with Jan Born and his group, Medical University of Luebeck, we run a study “Dimensional complexity and power spectral measures of the EEG during functional versus predicative problem solving” (Mölle et al., in print). The EEG was recorded in 22 young men (Fig. 4; students at the Medical University of Luebeck) while solving QuaDiPf tasks.

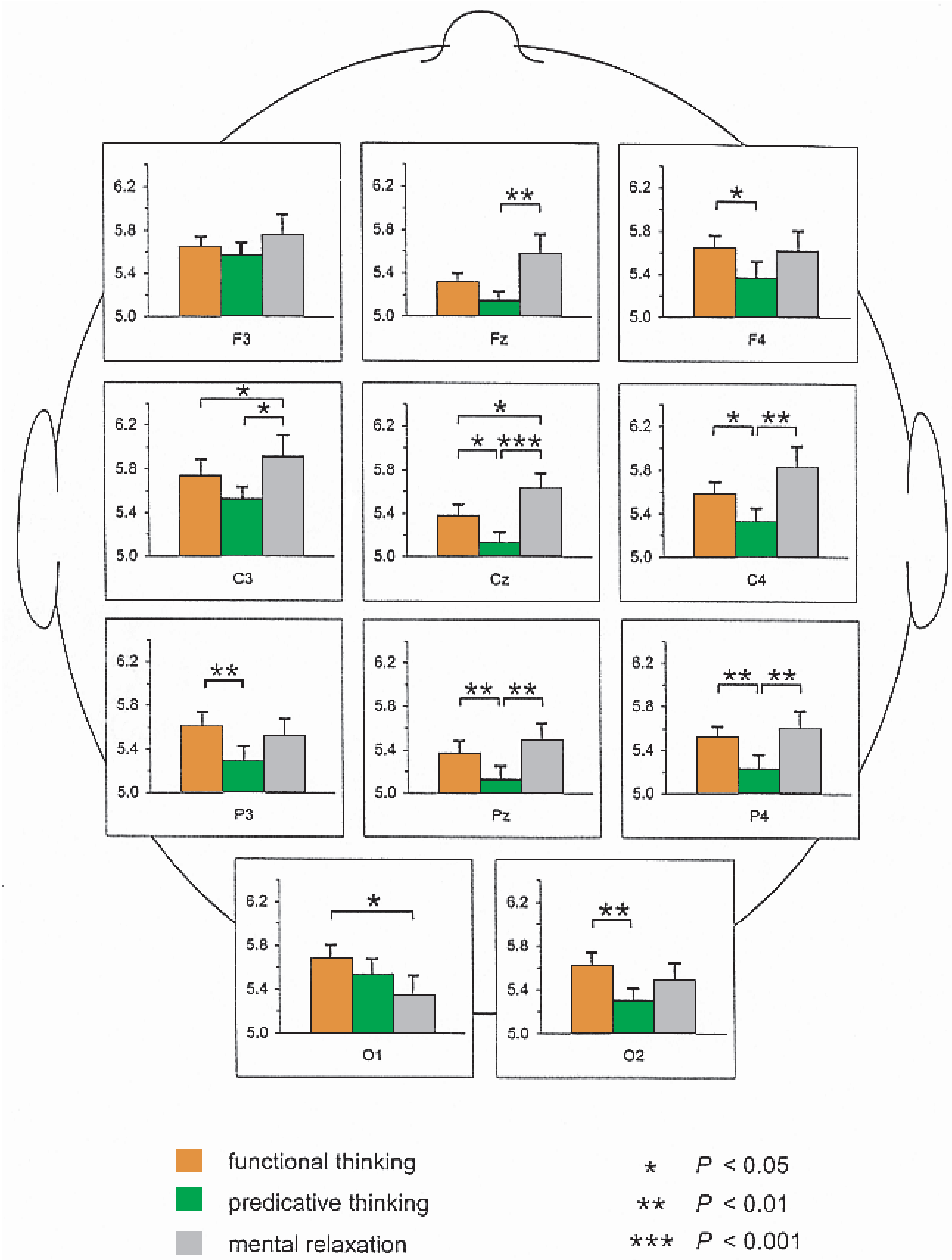

Fig. 5: Results of the EEG-Study (cf. Mölle et al., in print)

Because of known gender differences in brain activities, it was important to work with subjects of equal sex. We decided to start with male subjects, hoping that among them will be enough typical thinkers in both modes. It turned out, that we were lucky in the predicative case, but not the like in the functional case. Familiar with the behaviour of the students in our department of mathematics and computer science in Osnabrueck, the Osnabrueck staff was quite a bit astonished, that the young medical men didn't show up so much an ability to good functional reflections. Further experiments have to follow. In this first experiment the subjects performed on three different blocks each including 4 QuaDiPF tasks. After having completed a pattern mentally by his own the subject had to draw his solution and to explain why it fits the pattern well (Situation in Fig. 4). In the first block the subjects were asked to solve the tasks spontaneously. In the second and third block the subjects were primed to do it in a functional or a predicative way respectively. The priming took place during three tasks, for that purpose a typical functional or predicative argumentation for the solution was presented. The subject was asked to solve the fourth task just in the way it had been shown to him. The EEG during thinking on the fourth task of each condition was taken for analysis. The EEG complexity during predicative thinking decreased in comparison to functional thinking and mental relaxation, with this reduction being most pronounced over the right and parietal cortex. A reduction in dimensional complexity during functional thinking as compared to mental relaxation which was concentrated over the left central cortex, although significant, was less clear (Fig. 5).

5. Eye Movement - Reflections

Especially tasks like 3c put some problems in adequate handling. Carpenter et al (1990) tried to analyse the manner how subjects solve the Raven Matrices using eye-tracking methods. They rely on a (predicative) classification of the Raven tasks, which worked except for one task (APM No. 18 / isomorphic to fig. 3c): "*Problem was not classifiable by our taxonomy*" (Carpenter et al. 1990, p. 431). This very task No. 18 is in its style unique in the APM-Test. By means of an added functional classification on one hand such "mysterious" tasks could be approached systematically as well and on the other hand concerning tasks with a "double" nature like fig. 3a there could be offered, additionally to the predicative classification, a functional classification. Therewith a broader understanding of the subjects cognitive behaviour is possible.

In the near future we will run eye-tracker experiments using QuaDiPF tasks. We are convinced, that the results will support the theory of functional/predicative thinking. We expect, that we will found different patterns of eye-movements which are either specific for a predicative analysis or a functional analysis: Like in Carpenter et al. (1990) we should find eye-movements following essential properties (Fig. 6a), but moreover we should find eye-movements along the production process of the step by step developing states of specific objects (Fig. 6b).

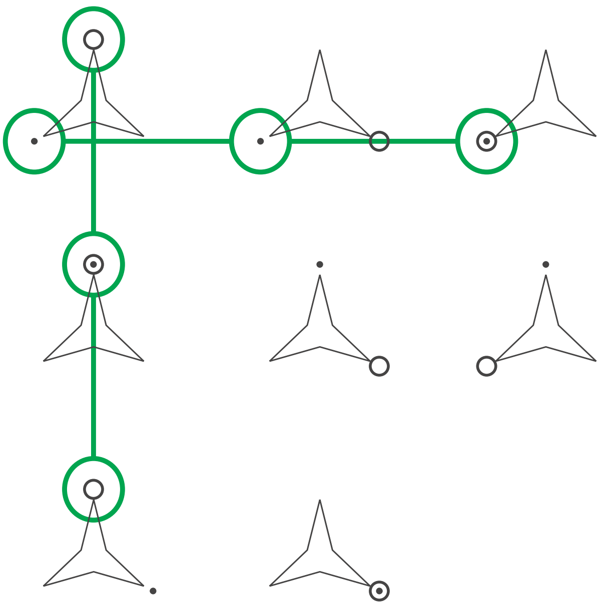

Fig. 6a: Eye movement during predicative analysis

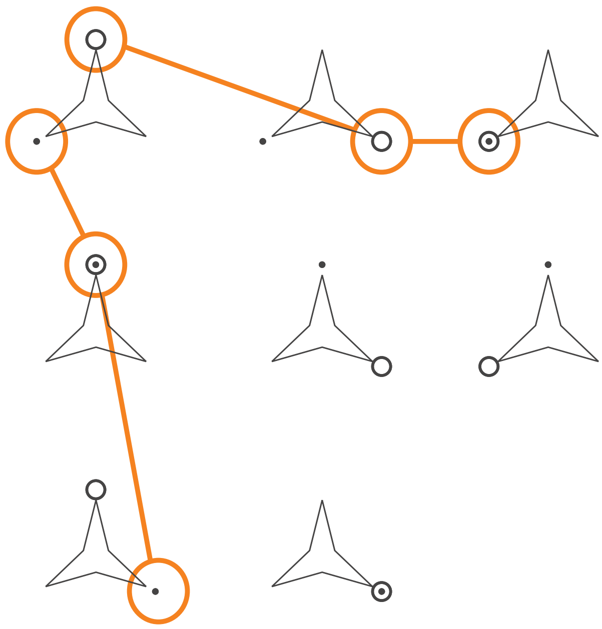

Fig. 6b: Eye movement during functional analysis

6. Outlook

When we started to run our research work we could show that the distinction between functional versus predicative thinking is useful to analyse the behaviour of subjects in problem solving situations in the field of mathematics and computer programming. Viable concepts (in the sense of von Glasersfeld 1995) there can be created in one or the other manner. Later on we could even focus on the nature of these two thinking modes using much simpler tasks of a nonverbal intelligence test, which turned out to be usable in EEG-measurement-environments.

In our days we still see the problem that it is much easier to communicate predicative ideas than functional ones. We agree with Vandamme, that the problem will be to find more adequate representations for functional ideas (Vandamme calls them: action oriented, which we don't like so much because there do exist predicative actions, e.g. in solving a jigsaw puzzles and we want to stress the van der Waerden aspect of motoric thinking in terms of construction instructions). We are convinced, that the new technologies which enable to create virtual and augmented reality (e.g. Vandamme & Morel 1996) and which bring up new tools to easily create and manage dynamic actions on the computer screen are actually the appropriate means to master this challenge.

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