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How Do Pre-/In-Service Mathematics Teachers Reason for or against the Use of Digital Technology in Teaching?

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Abstract: The role and the availability of digital technology in society is growing, which is why educators need to increasingly more often decide which types of digital technology to integrate into their teaching and when to integrate them. Thus, those decision-making skills need to be developed and measured especially for pre-service mathematics teachers. Therefore, we conducted an explorative interview study to understand the portfolio of argumentation on whether to use digital technology in different teaching phases and what criteria are used when making those decisions. Our results are based on ten interviews with pre- and in-service mathematics teachers in Germany. The analysis shows that (1) different levels of argumentation can be distinguished and (2) there are indications that teachers need to be aware of digital technology when deciding whether or not to use digital technology in a teaching setting. In addition, (3) besides the teaching-phase perspective and the learner-perspective, we expanded current research by formalizing the educator-perspective in a list of decision criteria. The compiled list of decision criteria was theoretically validated through the literature. In combination with the applied teaching phase framework and taxonomy of digital technology the list could aid in the development of the decision-making skills and potentially could result in a more reflective use of digital technology by pre-service and in-service teachers.

Keywords: digital competencies; pre-service teachers; student teacher evaluation; technological advancement; decision-making skills; mathematics teacher education

MSC: 97B50

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1. Introduction

Given the growing role of technology in society and education, as well as the growing number of digital technologies [1] available to teachers [1–5], it is important that the technology-related competencies of educators constantly evolve [6]. Educators must decide increasingly more often which types of technology to integrate into their teaching practice and when to integrate them to be effective and meet the demands of society and their learners, not only because of curricular guidelines/standards, but also because of their intention to prepare their students for a work environment in a future that will be even more digitalized and driven by technology [7]. In summary, the crucial skills [8] and the knowledge [9] of educators for making appropriate decisions on when and which types of digital technology to use in teaching are driven by the increased digitalization of society and by the increasing number of digital technologies available.

Unsurprisingly, the skills and knowledge required for selecting suitable digital technologies have been added to educator competency frameworks such as the ISTE Standards for Educators [10] or the DigCompEdu [11]. The latter entails twenty-two competencies and one of them is "Selecting digital resources," which is defined as seeking "To identify, assess and select digital resources for teaching and learning. To consider the specific learning objective, context, pedagogical approach, and learner group, when selecting digital resources and planning their use." [11] (p. 20). The framework does not further outline the selection or decision process beyond the definition that is required for the development and the assessment of the competency "Selecting digital resources". However, in addition to the factors cited in the framework, namely the learning objective, teaching context, pedagogical approach, and learner group, other factors potentially play a role when educators are making those decisions [11–14]. They need to be understood to foster and assess such skills and knowledge; therefore, we conducted an interview study with pre- and in-service mathematics teachers to explore their decisions regarding their use or non-use of digital technology in teaching.

In the following sections, we first describe the applied definition of digital technology—which is an extension of "digital resources" as used in the definition of "Selecting digital resources" in DigCompEdu [11]—and the theoretical model for describing the teaching context. Then, we situate "Selecting digital resources" within a definition of educators' competencies and specify our research questions. Subsequently, we present the results of the interviews conducted in the time from April to July 2021 with ten pre-/in-service mathematics teachers from Germany.

2. Theoretical Background

Choices regarding the use or non-use of digital technology (dT) in teaching settings cannot be taken generally [15] but depend on various factors [11–13], including the teaching setting or teaching context and the type of dT. Thus, we introduce the model of teaching phases by Prediger et al. [16] to describe the teaching context and the taxonomy by Clark-Wilson et al. [1] for a description of dT.

2.1. Digital Technology in Education

Given the dynamic nature and growing number of dTs, a taxonomy for defining dT that provides a level of abstraction is needed. Different taxonomies have been developed over time to define and describe the use of dT in teaching mathematics. One of the early taxonomies by Schoenfeld [17] entails describing the potential use cases for computers in mathematics by distinguishing the aspects of "drill-and-practice, tools to do the drudgework, multiple representations, simulations, dynamic representations programming, and intelligent tutor systems". Other taxonomies [18] (p. 243) use a learner's perspective and their interaction with learning content using dT—Do, See, Read, and Learn, or a grouping of dTs by discerning the ways in which they are shaping the mathematical cognition—dynamic and graphical tools, tools that outsource processing power, new representational infrastructures, and the implications of high bandwidth connectivity on the nature of mathematical activity [19]. A taxonomy specifically for mathematical analysis software (MAS) by Pierce and Stacey [20] maps out the pedagogical opportunities of MAS by subject, classroom, and task. Further perspectives and a finer-grained discrimination are introduced in a taxonomy by Bray and Tangney [21] (p. 263), which distinguishes the categories of dT, learning theory, the Substitution Augmentation Modification Redefinition (SMAR) level [22], and purpose, each with additional subcategories.

The taxonomy by Clark-Wilson et al. [1] (pp. 1225–1226) groups dT by its use within a teaching situation, including the non-teaching related duties of educators [23,24]; the uses are termed as follows.

- *Organizing*: "As a support for the organization of the teacher's work (producing worksheets, keeping grades)";
- *Representation*: "As support for new ways of doing and representing mathematics";
- *Collaboration*: "As a support for connecting, organizing in communities, communicating and sharing materials";

- *Independent*: "…a commercial and industry driven function, which supports students' more independent work and focuses on practicing and assessing previously taught mathematical knowledge and skills in a range of online formats."

Whereas the other taxonomies of dT in education focus more on technology relative to the mathematical learning content, the taxonomy by Clark-Wilson et al. [1] also includes the aspect of technology aiding educators in organizing their work. This aspect is also emphasized in teacher competency and knowledge frameworks [9] (p. 1028), [11] (p. 19); therefore we used this taxonomy for our research.

Digital technologies such as software applications or hardware can be assigned to each of the four technology groups, by either a specific application name or a generic term for a particular technology. Computer programs such as Excel, Numbers, and Google Sheets would be examples of the former, which are all referred to as spreadsheet software. Further, when categorizing dT using the taxonomy, some dTs may only provide a subset of the functionality described for a group whereas others fulfill the functionality of multiple groups. In addition, software applications can be further differentiated by open-source software, supported and developed by a community or made available commercially, and supported applications. Figure 1 shows samples using the generic terms for dTs and their positioning within the four technology groups—*Organizing*, *Representation*, *Collaboration*, and *Independent*. The shading indicates the degree of fit of a particular technology within a group.

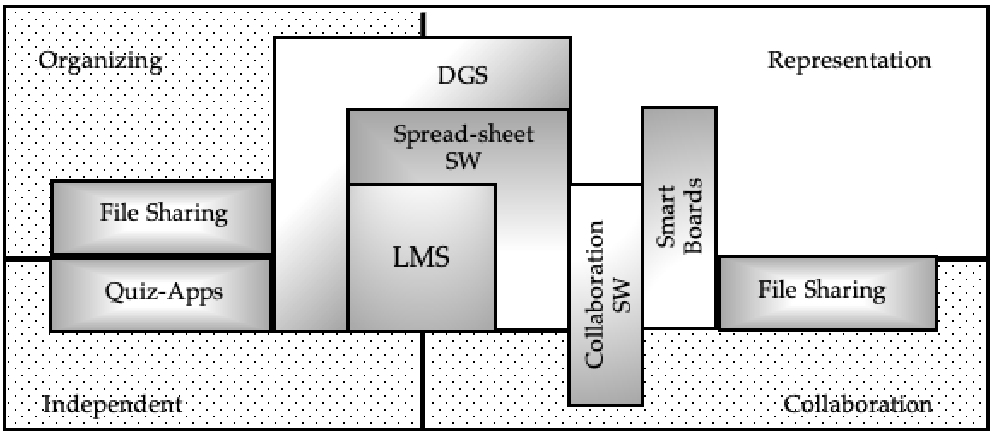

Figure 1. Examples of dT by the four technology groups *Organizing, Representation, Collaboration,* and *Independent*.

For instance, the Learning Management Systems (LMS) in the center entail the features or allow for the integration of technology in a way that enables all the aspects of the four technology groups to be addressed [25]. Spreadsheet software enables educators to track grades and fits the *Organizing* group. In addition, in mathematics education, spreadsheet software supports the creation of worksheets and the representation of mathematical learning content [26,27]; therefore, it also belongs to the *Representation* technology group. Dynamic Geometry Software (DGS) [28] is most applicable to mathematics as well as spreadsheets SW, whereas the technologies in the groups *Collaboration*, *Independent,* and *Organizing* are also applicable to other teaching domains. Next to the overlapping functions and features of the particular technologies, it needs to be noted, especially for software applications, that the feature set of a particular application can change over time.

First, the provided sample technologies and explanations highlight the complexities and demonstrate that a particular technology can be used in various ways in education. Second, the taxonomy encompasses dT related to teaching and the non-teaching related activities of educators. For pre- and in-service teachers to reflectively and effectively decide whether or not to use dT, they first need to be aware of the digital technologies that are available to them; therefore, an awareness of the technology of all four groups—*Organizing, Representation, Collaboration*, and *Independent*—is required knowledge.

2.2. Core Teaching Phases

The decision on the use or non-use of dT is correlated with the factors of learner-age and learning content, which are dependent on the teaching setting and teaching context. Prediger et al. [16] with supplements from Leuders [29] describe a model of teaching phases—based on the seminal work by Herbart [30]—that can be used to design teaching arrangements and to make situational teaching judgment calls. In the model, four core phases are differentiated and defined by their didactic function, cognitive activity, and epistemological quality. The didactic function describes the perspective of the educator, and the cognitive activity designates the perspective of the learner. From an epistemological perspective, the quality of the cognitive processes triggered by the cognitive activities are of interest. A translation of the definition of each phase is as follows [16] (pp. 770–772).

- Connecting (C): Describes the phase in which educators assess previous (related) learning content, activate the learner's real-world conception of (new) learning content, and provide an advanced orientation of (new) learning content through inquiry. The learners' cognitive activities in this phase include remembering, expressing initial intuitions, encountering difficulties, and raising questions.
- Discovery (D): In this phase, new content or contexts are explored and the didactic goal is to build concepts, develop procedures, and elaborate contexts using problem-containing intentional situations and tasks. The learners solve problems and explore mathematical patterns and phenomena in mathematical and real-world scenarios. The epistemological quality is determined by the discovery of new mathematical content and thus possesses an unknown coherence for each learner.
- Systemizing (S): Systemizing is the phase where the learnings of individuals are linked together with the entire learning group. Individual insights are connected where applicable to mathematical theorems. A balance between convergence-generating constriction and individual activity is achieved by learner activities such as the mapping of learning content, supplementing examples, and the explaining of the learning content. From an epistemological quality, this phase focuses on the regularization of learning content and represents the nexus between individual learnings, the mathematic learning content, and the learning group as a whole.
- Practicing (P): The didactic goal—the educator's perspective—of this phase is to make learning content available over a long term by practice and repetition, whereby the cognitive activities are described as "Practicing skills, reflecting on concepts, examining structures, solving problems—all of these can be addressed in practice tasks that are accessible to both strong and weak students alike. This is also referred to as "natural" differentiation." [29] (p. 134). The epistemological quality is determined by the type of tasks—meaningful, open for discovery, self-differentiating, reflexive—and the task repetition, differentiation of levels, operative flexibility, and the required reflective understanding.

The authors state that other aspects such as creativity [31–33] and the collaborative exchange [14,34] can be developed within each phase by providing those stimulations. This resembles the dT group of *Collaboration* [1] as outlined in the previous section. In the context of this paper, we use the model to evaluate the decision processes regarding the use or non-use of dT within each phase. Digital technology can be used in all four teaching phases to either support a didactic function, cognitive activity, or both.

Next, we highlight some of the studies which have investigated the use of dT in teaching and their didactic opportunities and cognitive affordances. Some of the findings are specific to a teaching phase, whereas others apply to multiple or all teaching phases. Drijvers et al. [14] highlight the cognitive support that dT provides to learners in the exploration of learning content, the fostering of self-directed learning by self-assessment [14] (p. 17), [35], and the promotion of dialogue and collaboration when learning mathematical content with dT [14] (pp. 20–23). The meta-analysis by Hillmayr et al. [36] reviewed 92 empirical studies in the teaching domains of mathematics and science in lower and higher secondary education and compared the learning outcomes of teaching with and without dT. The findings showed that teaching with dT had a small positive statistically significant effect on learners' attitudes towards the subject taught, which speaks to the cognitive motivational aspect that the use of dT has on learners regardless of the teaching phase and regardless of dT. Volk et al. [37] cite similar results at the primary school level.

The study by Roschelle and Singleton [38] specifically highlights the cognitive and didactic affordance of graphic calculators in teaching: "Underlying these pedagogical affordances, we see several basic cognitive contributions of calculators to student learning. One prominent factor involves reducing cognitive load and allowing students to focus more attention on high-level thinking [39]. Students with calculators can take on traditional tasks in new ways and also tackle new topics that would otherwise be inaccessible" [38] (pp. 954–955). Similarly, the study by Barzel and Möller [40] highlights the motivational and cognitive support that graphic calculators provide when exploring learning content using trial and error methods, again underlining the use of dT in the *Exploring* phase. Equally, in the context of geometry and calculus, the use of dT increases the didactic possibilities of educators and the ability of learners to visually explore the learning content using dT[41], once again stressing the value of dT in the *Exploring* phase. The reduction of learners' cognitive load by outsourcing mathematical operations to a Computer Algebra System (CAS) and consequently freeing-up teaching time is emphasized by Peschek and Schneider [42]. This aspect is also not explicitly stated, but can apply to the phases of, *Connecting*, *Exploring*, and *Practicing*. It is to a lesser degree applicable to the *Systemizing* phase since the focus there is to ensure the individual learnings are linked together with the entire learning group.

The use of well-designed interactive digital learning environments with interactive and adaptive exercises with feedback lowers the cognitive load of learners and leads to better learning results than traditional paper and pen worksheets, especially for low-achieving learners [43]. This shows that dT in the form of eBooks can provide value in the teaching phases of *Connecting*, *Exploring,* and *Practicing*. Ziatdinov and Valles [44] highlight DGS's positive effect on learning outcomes at secondary and university levels in mathematics and STEM education, as well as the ability to use DGS for exploration in modelling tasks. Similar to in other studies [45,46] the motivational effect of DGS dynamic worksheets is cited. Thus, in the context of the teaching phase model DGS can be applied in the *Connecting*, *Exploring,* and *Practice* phase and as previously stated is part of the dT groups of *Organizing*, *Representation,* and *Independent*. Similar highlights were presented in the study by Lindenbauer and Lavicza [47] on the use of DGS worksheets for connecting new learning content to existing knowledge and experiences, which is thereby an example of the use of dT in the *Connecting* phase.

The provided empirical studies and theoretical papers are just some examples of different dTs and their didactic and cognitive affordances in primary, lower and higher secondary, and higher education in the context of mathematics education. While some of the studies are directly attributed to a particular phase, others apply to all or multiple phases.

2.3. Digital Technology Affordances for Educators

Besides the didactic possibilities of dT and its affordances for supporting the cognitive activities of learners, the use of dT also presents affordances to educators themselves. In a study by McCulloch et al. [12], a participant highlighted the benefit of a quiz-software—from the dT of the group *Independent* in the taxonomy described in Section 2.1—as follows: "So, Kahoot, I like to use it as a review game...and it's nice for me as a teacher because I immediately get the results. And so, we can see how many people got it right, and how many people got it wrong" [12] (p. 34). The aspect of immediate feedback is more formally described by Drijvers et al. in the context of dT for assessment: "The scoring argument: Grading of students' work may be automatized. This may save much time for the teacher. In addition, automated grading is not only fast but also objective and consistent; its results may provide data for learning analytics." [14] (p. 13). Both instances speak to the properties of educators' efficiency and time saving. The constraints of such dT are also notable because of the level of effort involved and the ease of the use of the technology [14] (pp. 13–14). In addition, the assessment and quiz software, as with any other technology used in education, need to meet the local data protection and privacy laws [48]. Educators need to take those requirements into consideration when employing dT in their teaching [9] (p. 1039), [11] (p. 20), [2] (p. 312). The notion of the level of effort involved for educators when educating themselves on dT versus the merit of dT is captured in the technology adoption model for mathematics by Tatnall [13] (pp. 1206–1207). The model describes the decision to use dT as being contingent on the performance expectancy (PE), the effort expectancy (EE), social influence (SI), and facilitating conditions (FC) assuming the adopter has free choice in the decision. The studies by Al-zboon et al. [49], Birch and Irvine [50], and Chao [51] highlight those factors when adopting dT in education.

To recapitulate, in Section 2.1 we have described the taxonomy of dT [1] and the reason for its application in the study. The applied definition of dT in the form of the taxonomy in this study is an extension of the term "digital resources" as defined in the framework by Redecker et al. [11] (p. 90) and consequently we refer from here on to "Selecting digital technology" as opposed to "Selecting digital resources." In Section 2.2, we have introduced a model to describe the teaching context in the form of the teaching phases—*Connecting*, *Exploring*, *Systemizing*, and *Practicing*, which are different from the learning objective and learner group—applicable to the decision and selection process. In Section 2.3, we have cited factors that might influence educators' decision of whether to use dT because of their merits and the involved effort for the educator. We also want to point out that in the literature, there are different definitions of the term competency [52,53] suggesting that competency entails knowledge, attitude, action, and skills [8]. We consider "Selecting digital technology" as one of the skills educators require to successfully teach with and take full advantage of dT in education and therefore refer to it as a skill and not a competency as stated in the DigCompEdu framework [11].

3. Research Objectives

To understand the selection and decision process regarding the use of dT in teaching mathematics (as a part of the digital competencies of (prospective) teachers), we posed the following research questions (RQs).

RQ1: How do pre- and in-service teachers reason for or against the use of dT in different teaching phases?

This question will be specified in the context of a specific learning subject, learner-age group, and teaching phases (i.e., all four phases of the model by Prediger et al. [16] are addressed) and represents the teaching-phase perspective.

RQ2: How do learner-age and learning content factor into the decision of pre- and in-service teachers when deciding on the use of dT in teaching?

With this question, we specifically inquire about the learner-age and at what age to start teaching with dT, as well as the learning content. These criteria in the decision process denote the learner's perspective.

RQ3: How do the affordances of dT for the educator factor into the decision of pre- and in-service teachers when deciding whether to use dT in teaching?

Here, we want to explore the aspect of the educator's efficiency and the level of effort for educators associated with dT as outlined in Section 2.3 and any other factor participants would consider when making the decision. The RQs segregate the decision by three perspectives: teaching-phase—learner-age/content-perspective, and educator-perspective.

4. Materials and Methods

In the following sections, we provide an overview of the research design of the explorative interview study, including the participant selection process.

4.1. Design of the Explorative Interview Study

To answer our research questions, we have used semi-structured interviews with pre- and in-service teachers (i.e., novices and experts) regarding the use of dT in mathematics lessons. Along with some demographic questions, the respondents were asked to explain whether they use dT in the four phases of teaching as well as specific questions defining the mathematics subject and the learner group. In this part of the interview, we let the interviewees choose their preferred mathematics subject and learning group to reason on familiar grounds. In the second part of the interview, we inquired on the use of dT in teaching on a more general level—not restricted to a particular subject and student group. In particular, we inquired how the learner-age and the learning content factored into the decision on whether to use dT and gave room for the interviewee to cite any other factors they consider when making that determination. This two-pronged approach was taken to gain an understanding of how participants would reason within a specific setting and in broader terms of teaching with dT.

4.2. Participants

To obtain an understanding of the varying argumentation on whether to use dT and the argumentation used in the reasoning, we looked for participants with different degrees of experience.

- Pre-service mathematics teachers at the beginning of their university studies, who had limited exposure to didactic concepts and digital education technology.
- Pre-service mathematics teachers towards the end of their university studies with the theoretical didactic knowledge provided by the university curricula.
- In-service mathematics teachers with multiple years of teaching experience in lower and upper secondary and higher education to obtain a practical perspective.

For pre-service mathematics teachers, the school level was less relevant, especially for pre-service teachers at the beginning of their university education, as the curricula are identical. For in-service teachers, we looked particularly for teachers in lower and upper secondary and higher education, since at those levels, dT is part of the curricula in Germany. To obtain the perspective of how dT is integrated into education standards and when DGS environments are created, we looked for participants who were involved in those activities and decisions.

Our final participant sample consisted of two pre-service teachers at the beginning (1st and 2nd semester) and three pre-service teachers towards the end of their studies (6th, 8th, and 10th semester), who studied at a German university with an emphasis on special or on lower secondary education. The pre-service teachers for primary and special education receive the same mathematics university curriculum. Additionally, five German in-service mathematics teachers participated, whose teaching experiences varied between four and thirty years in lower and upper secondary education. None of the pre-service teachers had any formal training on using dT in education, whereas one of the in-service teachers participated in training programs on the use of dT in education; the others taught themselves by preparing teaching artifacts and teaching with dT. All in-service teachers used dT in their teaching—as it is mandated at the grade level they teach—and two of them were teaching university seminars for pre-service mathematics teachers on the use of dT in mathematics education. One of the in-service teachers was involved in the design of curricula for German mathematics education. He and one other in-service teacher also had experience with dT in mathematics education since its integration into schools, starting from the pocket calculator to the introduction of DGS. In addition, both were active in the creation of the tasks for the central upper education math exam and the creation of DGS worksheets. The broad diversity of participants was chosen to enable the maximum variability of answers and reasoning and to obtain the perspectives from different vantage points within education, such as pre-service teachers at the beginning and the end of their development, perspectives from practical teaching experience, creators of DGS work environments, and architects of lower and higher education curricula.

4.3. Data Analysis

Ten interviews were conducted between April and July 2021 via a video-conference tool and were transcribed. The transcripts were coded via qualitative content analysis [54] using MAXQDA [55]. In the following, the results of the coding are presented. Citations are abbreviated and designated as either pre-/in-service teacher indicated by the prefix "Pre-T-"/ "In-T-" followed by a number indicating the semesters of study or the years of teaching as well as the time stamp within the interview. The English translations of the interview coding by the authors are presented.

For research question RQ1 there is no objectively correct answer to the question of whether to use dT in teaching [15] (p. 14). Using a specific digital tool can be a good decision in one class, while it would be a bad decision in a parallel class. Therefore, we cannot rate decisions regarding the (non-)use of dT. However, we can evaluate the arguments used to back up this decision and determine whether they are didactically grounded or not. Thus, our analysis is two-layered, first taking note of the decision, and second analyzing the arguments used to explain this decision. For the latter layer, we differentiated argumentations with (a) no argument, (b) argument(s) which were not substantiated or were generic, and (c) argument(s) substantiated by either a didactic function, cognitive activity, or interviewee's own application.

For research questions RQ2 and RQ3 we took note of the learner-perspective criteria—age and content—and the educator-perspective used in the decision process and then categorized them into theoretical and teaching practice-based criteria. Theoretical criteria are supported by theoretical studies and findings, whereas teaching practice-based criteria speak to the practical teaching experience. Criteria which fit both descriptions are coded accordingly. We elaborate on the designation of theoretical and teaching practice-based criteria in Sections 5.4 and 5.5.

5. Interview Coding and Discussion of the Findings

In Section 5.1, we provide sample responses and the coding of those responses followed by Section 5.2 with a discussion of the results for RQ1—the teaching phase perspective. Similarly, in Sections 5.3 and 5.4 for RQ2—the learner perspective, and in Sections 5.5 and 5.6 for RQ3—the educator perspective.

5.1. Interview Coding Regarding RQ 1—Teaching-Phase Perspective

Two different directions within the first section of the interview were taken by the participants. The first direction was taken by one pre-service teacher in the sixth semester, who stated that the selection of dT cannot be made by the teaching phases and that the decision is rather based on the type of technology. This approach is similar to the taxonomies as outlined in Section 2.1, as it maps the features of dT to a didactic outcome and teaching situation. Even though this approach is valid, for the analysis, this interview was excluded as it doesn't answer the research question at hand. The second direction was taken by all the remaining participants, whose responses were used as the bases for answering the research question using a two-layered approach. In the first layer, we take note of the decision and analyze the arguments used to explain this decision. To that end, we first provide some examples of the applied coding system by teaching phase and the suggested dT within each phase, and later provide a summary of the types of argumentations clustered by the participant groups.

5.1.1. Coding of the Argumentation Used in the Connecting Phase

The following response is an example of a decision to not use dT in the *Connecting* phase because of a lack of knowledge of any applicable technology.

> [06:17]—Pre-T-2: *"Connecting. Well. I would say no…At the moment, I cannot think of a use for software to capture one's previous experience. I think that I would rather do in a conversation…"*

The method—conversation or plenary discussion—was suggested in the context of small size groups of special education learners. A response providing no argumentation but explaining the use of dT to connect to the learning goal (Symmetries in the 3rd grade) using an example is provided next.

> [05:06]—Pre-T-2: *"So one idea would be, for example…let them [the learners] google any symmetrical objects…if one wants to make it a bit more action-oriented, one could also send them [the learners] out and say: photograph anything symmetrical and discuss it afterward."*

The suggested dT—Google search and a digital camera—are examples of technology use and their potential to motivate learners.

A response with an argument substantiated by a didactic and cognitive purpose follows, which explains the use of a quiz app [Biparcours] versus paper and pen worksheets in the *Connecting* and *Exploring* phases.

> [11:16]—In-T-10: *"So now I use a digital tool [Biparcours], which is not explicitly mentioned in the curriculum… I found this methodology useful for motivation and to promote self-directed learning. The [students] can work through the questions in the Biparcours at their own pace and receive automated feedback and continue…"*

The app is used to reactivate the learner's prior knowledge of a related subject and let them explore the new content. The choice is explicitly supported by the didactic goal in those phases and the fostering of self-directed learning. In addition, the motivational aspect of technology with respect to the cognitive activities of the learners is mentioned in the reasoning. Implicit in this response is also the aspect of automated feedback and leveraging of the group instead of individual results assessing the outcomes of the phases. This speaks to the property of the educators' efficiency gained from the use of the technology. Notably, in this use case the dT connects the prior learning content to the new learning content, whereas in the former response the dT is used to connect the learning content to a real-world application.

5.1.2. Coding of the Argumentation Used in the Exploring Phase

The last response in the previous section was in regard to the *Connecting* and the *Exploring* phase and is not repeated here. The following responses are entirely in the context of the *Exploring* phase and a response using a cognitive argument is provided next.

> [07:45]—In-T-4: *"I think that when you are exploring and discovering new mathematical facts, the effort is very high… and if you can relieve this high cognitive hurdle that is involved in the discovery in some form by having the technology remove certain repetitive sequences of actions, such as the construction, which is done identically over and over again, we can outsource that…"*

The argumentation highlights the aspect of outsourcing and reducing the cognitive load when introducing new learning content using dT. Another response is within the context of geometry, but now emphasizing the time saved using dT, which reads as follows:

> [08:13]—Pre-T-10: *"…when you conceive a hypothesis…and then you cannot test it that quickly [using pen and paper]. Or, in general, when using trial and error it is often much easier and faster with digital tools, especially when it comes to dynamic geometry software. For example, that means you do not waste time on any elaborate drawings …"*

The argumentation explicitly speaks to the time saved for learners, but a similar benefit can be assumed for educators. An argumentation of a pre-service teacher in the second semester reads similar to this:

> [07:13]—Pre-T-2: *"Yes…I have only just familiarized myself with GeoGebra this semester. At some point afterwards, that was then quite practical, for finding things out, for exploring. Somewhat similar to discovering how things relate to each other? That becomes difficult at some point if you do it with a pencil and paper, and then it becomes static…I thought that it was quite good [using dT] … because it is actually quick and you could play around."*

This participant is in favor of GeoGebra—a type of DGS—in the context of geometry and the *Exploring* phase, because of his own experience using DGS at university.

5.1.3. Coding of the Argumentation Used in the Systemizing Phase

First, a response of a pre-service teacher who understands the sole purpose of the *Systemizing* phase to connect the learning content to a mathematical theorem, but not necessarily the aspect of ensuring that the education of the individual is shared across the entire learning group.

> [06:20]—Pre-T-2: *"Yes, let us systematize, e.g., … I distinguished point and axis symmetries and that could be systematized, e.g., … you could, for example, mirror the axis horizontally or vertically…exactly this could be systemized"*

In addition, rather than providing an argument for the decision to use dT in this phase, an explanation of how it would be conducted is given and the explanation potentially indicates an incorrect understanding of the mathematical theorems. Another pre-service teacher stated to not know any dT applicable to this phase.

> [14:25]—Pre-T-10: *"… My problem is somewhat that I do not know what kind of possibilities there are because I do not know [applicable dT]…"*

The response of an in-service teacher regarding the learning content of symmetries highlights the use of dT to enhance communication and collaboration—suggesting the use of dT corresponding to the *Independent* group.

> [15:17]—In-T-4: *"…of course, you can also use digital tools to promote communication. Online platforms, forums, or digital whiteboards, on which one has the possibility, especially now in times of distance learning, to share [results and findings] and to enter into discussion with each other. So, these are possibilities enabled by digital tools."*

The response with the explanation of how dT would be integrated is discounted in the final results.

5.1.4. Coding of the Argumentation Used in the Practice Phase

In regard to the *Practice* phase, an in-service teacher argues for the use of paper and pen worksheets rather than dT so that the learners have their worksheets for later reference. The Quiz-App, Biparcours, which this in-service teacher used in the *Connecting* and *Exploring* phase, is not used in the *Practice* phase because it would require learners to register to keep a permanent record of their worksheets. Due to data privacy concerns and the registration requirement, the dT is not used.

> [13:33]—In-T-10: *"…there is a [paper] worksheet for this, which can be used for practice, because in the Biparcours the results are not permanently stored…since the students do not use a login… they cannot [go back at a later point] to look at things…"*

An instance of a response with a very generic argument by a pre-service teacher for this phase reads similar to this.

> [09:38]—Pre-T-2: *"So first of all, I think that it generally makes sense to use it [dT] because I just think that it totally focuses the attention of students."*

This very generic response was excluded from the final list of arguments.

5.1.5. Summary of the Arguments by Teaching Phase

In the previous sections, we provided examples of the argumentation used by the participants for or against the use of dT in a teaching setting in relation to *Connecting, Exploring, Systemizing*, and *Practice*. Figure 2 shows a summary of the arguments used by the participants for or against the use of dT for each teaching phase. The arguments supporting a didactic aim are shaded dark grey and those supporting the cognitive activities of learners are shaded light grey, and a lighter grey is used when the participant's own experience of using the dT was used in the argumentation. Only arguments that are substantiated are included by an abbreviation in the figure. Choices that were not substantiated or that were too generic are not shown. From Figure 2, the observation can be made that participants provided arguments for or against the use of dT and that a portfolio of arguments exists. Another observation is that in the *Exploring* and *Practicing* phase the decisions that are supported by an argument are unanimous, and in the *Connecting* and *Systemizing* phase a prevalent reason for not reasoning in favor of dT is an unawareness of an applicable dT. Further discussion of these results, particularly the unawareness of an applicable dT, is deferred to Sections 5.2.3 and 5.2.4. Notably, none of the participants cite a poor teaching experience with dT as a reason for not using dT in a particular teaching phase.

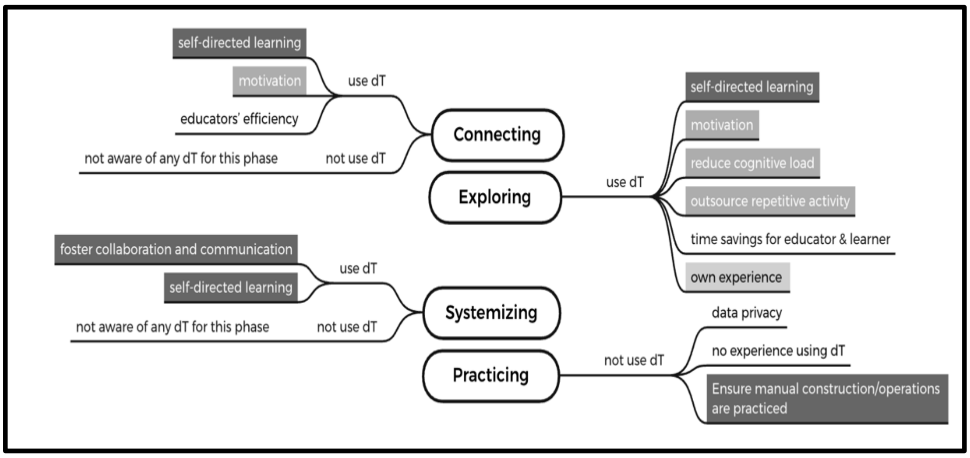

Figure 2. Arguments for or against the use of dT for each teaching phase.

We also coded the dT cited by the participants in the context of each teaching phase in their reasoning. The results of the three groups of participants, Pre-T-2, Pre-T-8/10, and In-T-x, and for each participant are shown in Table 1. The dT groups as defined in Section 2.1 are used, whereby the letters in each cell correspond to the dT groups—*Organizing* (O), *Representation* (R), *Collaboration* (C), and *Independent* (I)—and a dash if no specific dT was mentioned or if the dT was used in generic terms. The expression "no dT" indicates that the participants argued not to use dT for didactic reasons unless noted otherwise. In addition, we distinguished for each teaching phase if the dT was cited in connection with a cognitive (c) or didactic (d) argument. Exceptions to the two categories of argumentation are noted within the footnotes of the table.

Table 1. Digital technology named for each phase by the participants.

Part. Group		**Pre-T-2**		**Pre-T-6/8/10**			**In-T-x**				
Part. No.	**Arg.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Connecting	c	*	-	Excluded (see below)	-	-	-	I, O	-	-	-
	d	-	-		-	-	-	I, O	-	-	-
Exploring	c	*	R **		R	R	R	I, O, R	R	R, I	R
	d	-	-		-	-	R, O	I, O, R	R	R	R
Systemizing	c	-	-		-	-	-	C	-	R	-
	d	-	-		no dT	I ***	-	C	-	R	no dT
Practice	c	-	-		-	-	-	-	-	-	-
	d	-	-		no dT	no dT	no dT	no dT	no dT ****	no dT	no dT

Column "Arg." indicates if a dT was mentioned in connection with a cognitive (c) or didactic (d) argument. Participant 3 was excluded for disagreement with the notion of teaching phases in the decision as queried in RQ1. * Mentioned Google search and digital camera. ** dT was cited because of a recognition of the merits of the participant's personal usage *** Mentioned the web app "number-line" in context of self-directed learning. **** stated to not use dT because of no experience using dT in this phase

There are indications of multiple patterns in the table. For one, one Pre-T-2 named only technologies in the *Representation* group (and here also only GeoGebra) but was able to argue for their use in the *Exploring* phase such as Pre-T-8/10 and In-T-x. However, in the latter group there is one participant who did not only provide the most comprehensive list of technologies but also was able to identify and argue for the use of dT regarding the *Independent* and *Organizing* group in this phase. An indication of another pattern is that in the group of In-T-x, participants provided didactic and cognitive arguments for the use of dT, unlike the Pre-T-x who provided either didactic or cognitive arguments. We further discuss these patterns in particular for the *Exploring* and *Practice* phase in Sections 5.2.1 and 5.2.2.

5.2. Discussion of the Findings Regarding RQ 1—Teaching-Phase Perspective

In regard to RQ1 "How do pre- and in-service teachers reason for or against the use of dT in different teaching phases?", we first analyzed the responses for the *Exploring* phase, in which all the participants elected to use dT, and then at the *Practice* phase in which none of the participants elected to apply dT. Subsequently, we detailed the argument of "not being aware" of any specific dT for a teaching phase; then, we extended the analysis of the argument of "not being aware" to dT of the group *Organizing* beyond the teaching phases.

5.2.1. To Use dT in the Exploring Phase

The coding of the participants' responses shows that all the participants unanimously decided on the use of dT in the *Exploring* phase. The arguments for the use of dT in this phase are in line with the previously cited empirical and theoretical studies in Section 2.2, namely [41,44,46] as well as [56,57]. The level of argumentation varied among participants, as can be seen in Table 1. Whereas some participants provided elaborate explanations citing didactic and cognitive reasons as well as the affordances of teacher efficiency, others provided either only a didactic or cognitive rationale in their explanation for this phase. Additionally of note is the coherence in the arguments by the participant groups—pre-service teachers and in-service teachers in this phase. The participant providing an argument for using dT because of a recognition of its value from prior use as part of a university course by a pre-service teacher at the beginning of his study is a reasonable response, given his second semester-level of education. The argument is also supported by studies on dT acceptance in teaching, citing the performance expectations of dT as one of the factors in the adoption of the technology in teaching practice [49,50]. As elucidated in Section 2.3, these studies approach the decision of the use or non-use of dT from the perspective of dT adoption with the assumption that the decision is a free choice, which is not always the case [13] (p. 1207). In our study, we focus—regardless of the free choice of dT—on the decision to use or not use a dT in a particular teaching situation. We agree with Tatnall [13] that not all decisions on whether to use dT in a teaching phase are a free choice, as some use of dT is mandated by the curriculum, but regardless of whether it is a free choice, the performance expectations of dT can be equated to the arguments in the decision process as summarized in Figure 2. Hence, the decision process of the use of dT in a particular teaching situation is indeed valuable even if the particular dT is not a free choice. Going through the decision process and arguing for the use of dT in a particular teaching phase will potentially enable (especially pre-service) teachers who have less teaching experience to more reflectively understand how to use dT. This aim was not only articulated by Kaspar et al. [58] but also in the competency framework cited in Section 1: "To identify, assess and select digital resources for teaching and learning. To consider the specific learning objective, context, pedagogical approach, and learner group, when selecting digital resources and planning their use." [11] (p. 20).

To summarize, the participants unanimously agreed on the use of dT in the *Exploring* phase, but the level of argumentation varies. Regardless of whether the use of dT is a free choice, the decision process and argumentation for the use or non-use of dT potentially enables a reflective use of dT and the development of skills relating to "Selecting digital technology" [11].

5.2.2. To Not Use dT in the Practice Phase

In the *Practice* phase, none of the participants argued in favor of the use of dT. One of the reasons was not having experience applying dT in this phase, an aspect closely related to not knowing any specific dT for a phase, which we further detail in Section 5.2.3. Another reason for not using dT in this phase was data privacy [2,21,59]. To keep permanent records of their worksheets and practice materials, learners need to have access to tablets and laptops at home and school and would potentially have to register with the dT of the group *Independent*. Unless these technologies comply with local privacy rules and do not collect learners' data for commercial reasons, they are not applicable for education settings.

It is peculiar that the value of dT in the group *Independent* is seen in the *Connecting* phase, but the same benefit is not seen or not articulated in the *Practice* phase, in particular concerning the opportunities of using dT for self-directed learning [14] (p. 17) or assessment [14] (pp. 11–20), [12] (p. 34), [60], and [61] (p. 844). A possible explanation is that the use of dT for assessments is not widespread and not necessarily the main didactic objective of the phase. The other reason for not exploiting dT in this phase is that the participants saw the value of practicing constructions using paper and pen in geometry and performing operations without the support of dT in arithmetic in the context of the learning groups addressed.

To recapitulate, in relation to the nearly complete lack of dT experience in this phase, the argument for not exploiting dT in this phase comprised limitations in the available digital infrastructure, resulting in privacy concerns and the value of practicing mathematical constructions and operations without the support of dT.

5.2.3. To Not Use dT Because of Not Knowing of Any Applicable dT

For the *Connecting* and the *Systemizing* phase, one pre- and one in-service teacher stated that they did not know any dT applicable to the phases or could not contextualize their familiar dT with the didactic aim of the phases and therefore elected not to use dT. This variance of knowledge or awareness of dT is also seen in the overall coding of dT by the teaching phases in Table 1. However, to assess whether to use dT in a teaching situation or for administrative duties, an awareness of the available technology is required. If one is not aware of or does not know how to use a particular technology, any argumentation relating to its use in a particular teaching situation is either very generic or potentially biased towards one's personal orientation towards dT. As outlined in Section 1.1 one does not necessarily need to know to which particular group a dT belongs to in the taxonomy [1], but pre- and in-service teachers should be aware of and know to a certain extent the currently available dT within each group. The more one knows about a particular dT, the more one knows about the affordances dT provides in a teaching phase. The fact that pre-service teachers, especially at the beginning of their development process, only know about dTs from the *Representation* group is not too surprising. They might have had exposure to dT from their own time as students and thus have first-hand experience with dT such as DGS, calculators, and spreadsheets. Depending on the usage, they may or may not have experienced them or other tools in the context of the *Collaboration* or *Independent* dT groups. Even if they had exposure, they may not attribute their use to the didactic intention of the *Independent* or *Collaboration* categories in a particular teaching phase.

5.2.4. Awareness of dT in the Organizing Group

Similar to the teaching phases, the issue of not being aware of dT can be assumed for the dT of *Organizing*—the creation of worksheets and the tracking of grades and other administrative activities of an educator. Pre-service teachers have little to no exposure from their own school experience to the administrative activities of educators [62]. As seen when deciding on the use of dT in a teaching phase, being aware of dT or the use of dT for that purpose requires an understanding of those education obligations, which unless one has been exposed to or has had them explained in the pre-service teacher development process, little knowledge should be expected. Especially the potential for time-saving and automation by dT for teachers is cited in literature [23,24].

Therefore, less awareness of dT relating to the *Organizing*, *Independent,* and *Collaboration* groups in pre-service teachers at the beginning of their studies compared to in-service teachers with multiple years of teaching experience—as summarized in Table 1—is plausible but would require localized quantitative studies for confirmation. An awareness of dT is required to enable a reflective decision to use or not use dT in a teaching phase and an analogous relationship can be assumed to be true for the dT of the group *Organizing* although we did not specifically inquire on that in this study.

5.3. Interview Coding Regarding RQ 2—Learner Perspective

In the second part of the interviews, we inquired about how the learner's age factored into the decision process and at what age a learner should start using dT in education, and how the learning content factored into the decision. First, we show some responses the participants used concerning age and then content; responses of all ten participants were included in the coding and the results.

5.3.1. Sample Responses in Regard to "Learner-age" and the Use of dT

We asked how learners' ages factor into whether they use dT when teaching. Again, we show some responses that are now not necessarily in the context of particular learning content or a particular dT. The responses were categorized as theoretical and practical teaching criteria. The response of an in-service teacher for lower and upper secondary education reads as follows:

> [33:12]—In-T-10: *"…In the fifth grade, I cannot assume that every student has a cell phone. In tenth grade or the upper secondary level, it is more likely…"*

A response such as that would be classified as a practical teaching-based criterion. A different response to the question at what age a learner starts with dT in education, without specifying which dT to start with, reads as follows.

> [15:04]—Pre-T-2: *"…maybe the kids that are going into first and second grade now… they will [complete the upper secondary education] in 12 years or 13 years… where do we stand then [as a society in regard to the use of dT]? …then it is much better to start early and teach children from the beginning [with dT] instead to make a sudden switch at some point…"*

This response is more indicative of a personal orientation towards the use of dT and was not classified. Yet another response states that the age of the learner at which to start using dT is a ruling to be made when creating the education standards. This kind of ruling is professed as the current generation deciding for the next and could be classified as a theoretical as well as practical teaching criterion.

> [28:25]—In-T-28: *"…that is one thing [when to start teaching using dT] that you just have to negotiate from generation to generation…"*

This and the other responses can be summarized by criteria based on the availability of the learners' [own] devices, such as laptops and tablets on the one hand, and highlight that the choice of when to start teaching with dT is a decision similar to any other decision on education standards and must be agreed upon by the current generation for the future generation. The former is a practical and the latter a practical and theorical teaching criterion. The criterion indicative of a personal orientation towards the use of dT was not included in the final list of criteria in Section 6.

5.3.2. Sample Responses in Regard to "Learner Content" and the Use of dT

We asked for any learning content participants would not consider the use of dT. Again, we show some responses that are now not necessarily in the context of a particular learning age or dT. The criteria were again classified by theoretical and practical teaching-based criteria. Now, we demonstrate a potential response indicating that no learning content would be excluded from teaching with dT. This and similar responses are on the one hand an indication of a personal orientation and on the other hand a type of practical teaching experience.

> [31:17]—In-T-10: *"…I find it more difficult to think about what content I would not use [dT]. No, I cannot think of anything where I would say that I would not use dT, I would rather say that you cannot work only with dT. So, for example…[geometric]constructions I think, you can do [it] very nicely with GeoGebra, but it's also important to create drawings by hand…"*

Here is a response that highlights that in the history of education standards choices have been made to reduce the mathematical learning content independent of dT. The ruling to use dT for particular learning content is no different from those rulings.

> [27:10]—In-T-28: *"…you must have those discussions…and things develop…that decisions are made differently today than in 20 or 30 years… if you look, there are creeping processes [in the development of education standards] that have existed for a long time, which have not been influenced by the computer and the graphic calculator. In the 70s, we still taught all the derivatives' rules [at upper secondary level] …"*

Other responses regarding the learning content speak to the dynamic representation of mathematical learning content and the different forms of representation of learning content, the reduction of cognitive load, and motivation, which are identical to the arguments used in Section 5.1; therefore, we do not provide sample responses here to avoid repeating ourselves.

The responses can be summarized by a personal orientation of the participants towards the affordances of dT for teaching any learning content, by citing the historical development of education standards, and by theoretical responses speaking to the affordances of the representation and cognitive load of dT. Similarly, as before, the criteria based on personal orientation were discounted in the final list of criteria in Section 6.

5.4. Discussion of the Findings Regarding RQ 2—Learner Perspective

In the interviews, we inquired about the participants' criteria when determining the use or non-use of dT; specifically, how the learner's age and the learning content factor into the decision. These criteria related to the decision are of particular interest because dT needs to support the cognitive activities of learners appropriate for their age and support the didactic aim of the learning content. In the following sections, we link the provided criteria of our participants to theoretical and empirical evidence for validation.

5.4.1. Criterion: "Learner-Age"

For the criterion learner-age, none of the participants provided a fixed starting age as to when to begin teaching with dT. However, they provided criteria concerning learner-age with respect to the availability of a learner's "owned" devices and the required oversight needed when using dT.

The differences in opinion on when to start and to what extent to teach using dT can be seen in various OECD documents. For one, the 2021 OECD report [63] (p. 344) shows that countries start with dedicated ICT education at different grade levels; similar results can be found in the OECD report on the use of dT for pre- and primary school education—for learner-ages 3 to 8—during the COVID-19 pandemic [64]. Here, 79% of the participating countries report the use of dT—in the form of distance learning and to distribute learning content—to maintain continuity as appropriate "to a great extent" or "moderate extent" for both pre- and primary levels of education [64] (pp. 24–25). These reports do not detail the didactic or cognitive perspective of the use of dT but show that there is no consensus regarding the learner-age at which to start incorporating dT in teaching. Further, similar to our interview participants, these reports highlight that the availability of digital resources is one of the major obstacle in the use of dT [64] (p. 38) [65] (p. 11). The study by Weinhandl et al. [66] (p. 8) highlights the availablity of a student's own devices at school and at home as one of the major obstacle percieved by educators. Studies analyzing the use of dT at the primary school level [31,67] or preschoolers' level [68–70] show the affordances of dT even at these early school levels, in addition to how teaching settings or tasks need to be designed for learners at those ages. This further substantiates that there is no defined leaner-age when one can start using dT in teaching mathematics. It is more important that the learner-age is considered in the design of dT and in the design of the tasks. For younger learners, educators need to provide more guidance to ensure dT is used towards the learning goal, as outlined in the criteria list in Section 6.

To recapitulate, the criterion of this study regarding learner-age, namely the availability of dT—devices such as tablets and laptops—is supported by the OECD reports. Other aspects in the context of learner-age are the required learner oversight and the design of the dT, which are also supported by other studies in literature.

5.4.2. Criterion: "Learner Content"

The participants in this study considered the use of dT for any learning content and further delineated the affordances to what extent dT enables or supports dynamic and different forms of representations and the ability to outsource repetitive activities, which are not the focus of the didactic goal. The aspects of dynamic representation and different forms of representation have been cited in Section 2.1 within the definition of the dT taxonomies as one of the affordances dT provides [17,19,20]. Hohenwarter [71] emphasizes multiple forms of representation—symbolic and iconic—as well as dynamic representation using DGS in teaching mathematics. Hohenwarter states: "In this sense, GeoGebra offers two different registers of representations with its graphics and algebra views on the same abstract mathematical object. As a dynamic mathematics software, GeoGebra provides the symbolic and iconic representations of mathematical objects in two connected views side by side in order to support visualization and the principle of interaction of representations" [71] (p. 14). The same applies to CAS [42] (p. 192). Reducing the cognitive load and the outsourcing of repetitive activities have been shown in several studies [36,38,43]. These factors are also highlighted by McCulloch et al. [12]. The aspect of increasing a learner's motivation through dT is highlighted in the studies specifically related to desmos [46], GeoGebra [44], graphic calculators [40], and various dT in [36,37].

The criteria provided by the participants regarding learning content—enabling or supporting dynamic and different forms of representations, the ability to outsource repetitive activities, and learner motivation—are validated by the affordances of dT described in the literature.

5.5. Interview Coding Regarding RQ 3—Educator Perspective

In addition to learner-age and learning content, we inquired in the second part of the interviews how the affordances of dT for the educator factored into the decision process, and the responses of all ten participants were included in the coding and the results. An example for the criterion of data privacy was already cited in Section 5.1.4, which elaborated on the responsibility of educators to ensure that data privacy and data protection laws are adhered to. This is especially important for dT, which is not mandated by the curriculum and not pre-screened by the school board or equivalent authorities. A criterion addressing the potential time saved as well as the level of effort required to generate teaching material using dT is provided below.

> [40:47]—In-T-10: *"…creating a good [Biparcours] is time-consuming, more time-consuming than creating a paper-worksheet. Preparing GeoGebra [worksheets] can be time-consuming, especially if you are not trained…it can also be very easy if you can find something suitable online…."*

Similarly, addressing the criterion of the level of effort and time investment required for educators to learn and use dT is cited by a pre-service teacher based on experiences in her practice semester.

> [21:52]—Pre-T-8: *"…I found that in my practice semester, there were very motivated teachers, but at some point, they said their time is limited and they could not catch up on [dT], and thus stuck with analog teaching…."*

A criterion relating to the use of dT to produce higher quality teaching material and for saving time in preparation through the reuse and delivery of the learning content in class is cited by an in-service teacher.

> [44:55]—In-T-10: *"…Yes, I use PowerPoint a lot…because it reduces my [workload] …if I make a proper PowerPoint [presentations], then I put a lot of work into it…when I repeatedly teach upper-level courses…then I can often reuse it and save time in [preparation and delivery] and it is fancier when it is visualized. I have also received feedback from students that it looks more professional…"*

The citation also includes positive feedback from the students on the quality of the material. The previous in-service teacher also formulated the criterion of balancing the level of effort for learning a dT and the added value provided in this next statement.

> [24:26]—In-T-10: *"…the broader criterion I need to ask [myself] is what kind of added value does this tool bring to my teaching?… Depending on the extend of the added value, if it is a small added value, I would need a tool that I do not have to invest a lot of time with, one that is easy to use, and that does not require a lot of effort. When there is a large added value, I would also be investing more time in becoming more familiar [with the tool]"*

To summarize from an educator's perspective, firstly, one criterion comprises data privacy and data protection, and secondly, another comprises the potential time saved in preparation as well as the production of higher quality content using dT. The third criterion in the decision is the level of effort required to use and teach with a dT effectively.

5.6. Discussion of the Findings Regarding RQ 3—Educator Perspective

Regarding the educator perspective, a participant provided data privacy [2] as one of the factors he used when deciding on the use or non-use of dT. In particular, technology from commercial providers often places a higher burden on the user either because of financial costs, login and registration requirements, or the data collection by the provider. This is a factor educators need to consider when selecting a dT intended to be used by learners, and it has been cited in Section 2.3.

The factor of a teacher's efficiency towards quickly creating and rapidly modifying teaching artifacts using dT is similar to the argumentation in Section 5.2.1 and the affordances of the technologies for learners—namely outsourcing repetitive activities and using time for more high-level thinking tasks [38,42]. Using dT for assessments and providing educators with summative views of the results and generating automated learner feedback speaks to the time saved by dT [12]. These criteria can be mapped to the dimension of "Performance Expectation" in the cited technology adoption model [13]. The educator perspective—especially the aspect of teachers' efficiency—in the decision to use or not use dT was mentioned but not further formalized in the study by McCulloch et al. [12] (pp. 34–35).

Next to the time saved, educators need to invest time when learning how to use new dT themselves and they must factor that into their decision. The level of effort to learn a new dT needs to be balanced with the didactic value in teaching or the efficiency gains for the educator. This criterion aligns with the dimension of "Effort Expectancy" in the technology adoption model [13].

6. Summary of the Findings

Next, we provide a cumulative summary of all the criteria, regardless of the group that responded, organized by learner and educator perspectives. The criteria are formulated either as continuum statements (younger to older learners, less to more) or dichotomous yes/no statements. The criteria that are limited in their scope because of the context they were provided in or the participating teaching group being either special-, lower-, or higher education are marked accordingly.

Learner (age and abilities) perspective:

- The younger the learners, the lower the cognitive demand of the technology can be, which can be even less if the learner's do not own personal devices. In addition, the younger the learners, the more oversight is required to ensure that the dT is used responsibly and in the intended didactic manner.

Motoric abilities (special education):

- No: Do not use technology, if it is likely that it gets damaged.
- Yes: If it enables the inclusion of learners (overcomes the impairments of the learners).

Learning Content perspective:

- Yes: If the curriculum demands the use of a particular technology.
- Yes: If the content can only be taught using a particular technology.
- Digital technology should enable or support dynamic and different forms of representations and the ability to outsource repetitive activities that are not the focus of the didactic goal. *

Educator perspective:

- Technology by commercial providers often places a higher burden on the educator either because of financial costs, login and registration requirements, or the data collection by the provider.
- Teacher efficiency because of dT. The ability to quickly generate higher quality teaching artifacts and the ability to quickly modify teaching artifacts using dT.

- The level of effort an educator needs to invest to use or learn a dT, especially if it is a dT that is not mandated by the curriculum.

The criteria denoted with an asterisk and speaking to the aspect of cognitive load and different forms of representation are theory-based, whereas the others are teaching practice-based criteria. From the list, it becomes apparent that the participants used no hard rules as to what learner-age to start at for using dT. The statement in regard to the availability of learners' own devices is supported by the literature [72,73], but can only be seen as generic guidance and needs to be verified for each individual teaching situation and levels of access to the internet, which is a requirement for using some dT and depends to a great degree on the local circumstances [74] (p. 5), [75] (p. 36). The same holds true for the required oversight of learners when using dT. It should also be noted that some of the participants stated that they weigh their decision and selection of technology under different criteria and are making tradeoffs when deciding.

To summarize, the criteria defined in this section apply to the decision of whether to use dT in a teaching phase, but also for evaluating dT not previously used by an educator. The evaluation of dT closely aligns with the technology adoption model defined in the context of education [13]. The description of the criteria list in this section expresses the dimensions of the "Performance Expectation" and "Effort Expectations" one has when adopting a new dT [13]. The criterion speaking to the risk of damaging dT could be interpreted as what is labeled as an extension of the adoption model as a "perceived risk" or the risk using of dT [51]. The latter dimension is not as pronounced in the responses of the participants as the other dimensions, possibly because of the positive orientation of some of the participants towards dT, as seen in Section 5.3.2. Similar findings are seen in the study of Thurm and Barzel [76] (p. 57) concluding that in-service mathematics teachers assess the risks of teaching with dT with a lesser weight than beliefs about the potential benefits of teaching with technology (e.g., the support of multiple representations, the support of discovery learning)" which would support the findings in Section 5.3.2.

Secondly, the list of criteria in this section has been deduced from the participants' responses and is supported by the findings in the literature as shown in Sections 5.5 and 5.6. Therefore, it can be seen as a theoretically—but not empirically quantitative—validated list that could aid in the assessment of whether to use dT as well as when developing the skill of "Selecting digital technology".

In RQ1 we examined how pre- and in-service teachers argue for the use or non-use of dT in each teaching phase [16]. Closely related but in contrast to RQ1, in RQ2 we explored what criteria pre- and in-service teachers use regardless of the teaching phase in their decision, specifically learner-age/content—learner perspective—and in RQ3—the educator-perspective. Within our study, we did not inquire into the sequence concerning whether one first evaluates a dT regarding a teaching phase and then in regard to the criteria as listed in this section or vice versa. McCulloch et al. [12] (p. 30) suggest to first decide if dT should be used in a lesson and then the type of dT and lastly other considerations, which if translated to our study would suggest first assessing whether to use dT in a particular teaching phase (RQ1) and then considering the criteria summarized in this section (RQ2 and RQ3).

7. Conclusions

In the interview study, firstly, we were able to show how pre- and in-service teachers decide on the use or non-use of dT and how they reason for their decisions, differentiated by the four teaching phases. The choices vary by teaching phase, that is, in the *Exploring* phase, all participants unanimously decided on the use of dT, whereas in the *Practice* phase, all participants elected against the use of dT. In the *Connecting* and *Systemizing* phases, some participants decided for and some against the use of dT. The arguments to support the decisions to use or not use dT differ. This is especially the case for pre-service teachers at the beginning of their university studies, who in this sample provided more generic arguments, whereas in-service teachers supported their decisions by didactic, cognitive arguments. We cannot deduce from the results that the different level of argumentation applies to all pre-service and in-service teachers as shown in Table 1, only that different levels of argumentation exist. Quantitative studies in the population of interest would be required for that assessment.

Secondly, not being aware of or not knowing of specific dT applicable for a teaching phase inhibits the argumentation regardless of the teaching phase. Consequently, the participants defaulted to not using dT. This aspect is particularly important for the education of pre-service teachers and the development of the skill "Selecting digital technology." It is important that pre-service teachers become educated and exposed to dT of all four groups in the taxonomy [1]. To what extent they get trained on each particular technology needs to be balanced with the other developmental needs. We specifically inquired about the use or non-use of dT in the four teaching phases but dT can also aid educators in their administrative duties [23,24]. Here, it can also be hypothesized that because of the lack of knowledge of applicable dT for these duties, namely dTs of the group *Organizing* in the taxonomy [1], educators default to not using dT. Further quantitative studies are required to validate this claim.

Thirdly, the gathered list of criteria in Section 6 based on the responses of the participants combines theory and teaching practice. The criteria are validated by empirical and theoretical papers. We expand on the study by McCulloch et al. [12] by formalizing the "Educator Perspective" in the decision of using dT, especially the property of educators' efficiency. The list of criteria in Section 6 applies to the decision to use or not use dT in a teaching phase but can also be used when evaluating a new and previously unused dT. For the latter, the criteria, to some extent, express the dimensions of the technology adoption model for education termed by Tatnall [13].

This study is limited to pre-service teachers at one university and in-service teachers of one region in Germany, all in the context of teaching mathematics. Another limitation is the fact that in our study, we deliberately avoided focusing on a particular learning content, learner-age, or dT, and rather tried to understand how these factors influence the decision process. Therefore, our findings are intentionally broad and are applicable as a framework to evaluate the use or non-use of dT in the vast variety of different possible teaching situations. The list of decision criteria in Section 6 make them possibly especially valuable for pre-service teachers or anyone with less practical teaching experience, as it includes the responses of in-service teachers with practical teaching experience with dT and who teach on the subject of dT. To evaluate the effectiveness of the list of decision criteria studies with an objective assessment instrument would be required.

The learning content and the use of dT in education standards have changed over time and it can be assumed that with the availability of newer dT, the education standards will evolve further. An example of this tendency is the endorsement of using dT at the primary school level in Germany, which started in 2021 but did not suggest any specific dT [77] (pp. 91-93). Educators need to continuously reevaluate overtime when and in what teaching settings to use a given dT. In addition, regardless of whether the decision to use a particular dT comes from a teacher or from the requirements of the curriculum [13], the decision to use or not use a dT for a particular teaching situation is always the teacher's free decision. Consequently, there will always be a need to train pre- and in-service teachers in the skill of "Selecting digital technology." We suggest that the combination of the teaching phase model [16] and the dT taxonomy [1] provide an effective framework for developing the skill and can potentially result in a more reflective use of dT by pre-service and in-service teachers. However, quantitative studies would be required to substantiate these claims.

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References

1. Clark-Wilson, A.; Robutti, O.; Thomas, M. Teaching with Digital Technology. *ZDM Math. Educ.* **2020**, *52*, 1223–1242. https://doi.org/10.1007/s11858-020-01196-0.
2. Ally, M. Competency Profile of the Digital and Online Teacher in Future Education. *Int. Rev. Res. Open Distrib. Learn.* **2019**, *20*, 302–318. https://doi.org/10.19173/irrodl.v20i2.4206.
3. Diaz, P.; Ioannou, A. Learning in a DigitalWorld: An introduction. In *Learning in a Digital World: Perspective on Interactive Technologies for Formal and Informal Education*; Smart Computing and Intelligence; Díaz, P., Ioannou, A., Bhagat, K.K., Spector, J.M., Eds.; Springer Singapore: Singapore, 2019; pp. 1–12, ISBN 9789811382642.
4. Engelbrecht, J.; Llinares, S.; Borba, M.C. Transformation of the Mathematics Classroom with the Internet. *ZDM Math. Educ.* **2020**, *52*, 825–841. https://doi.org/10.1007/s11858-020-01176-4.
5. Valtonen, T.; Leppänen, U.; Hyypiä, M.; Sointu, E.; Smits, A.; Tondeur, J. Fresh Perspectives on TPACK: Pre-Service Teachers' Own Appraisal of Their Challenging and Confident TPACK Areas. *Educ. Inf. Technol.* **2020**, *25*, 2823–2842. https://doi.org/10.1007/s10639-019-10092-4.
6. Kaspar, K. Motivations for Social Distancing and App Use as Complementary Measures to Combat the COVID-19 Pandemic: Quantitative Survey Study. *J. Med. Internet Res.* **2020**, *22*, e21613. https://doi.org/10.2196/21613.
7. OECD. *The Future of Education and Skills Education 2030*; OECD: Paris, France, 2018.
8. OECD. *21st Century Skills and Competences for New Millennium Learners in OECD Countries*; OECD Education Working Papers; OECD: Paris, France, 2009; Volume 41.
9. Mishra, P.; Koehler, M.J. Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teach. Coll. Rec.* **2006**, *108*, 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x.
10. International Society for Technology in Education. *ISTE Standards for Educators: Computational Thinking Competencies*; International Society for Technology in Education: Washington, DC, USA, 2018.
11. Redecker, C.; Punie, Y. *Digital Competence of Educators*; Publications Office of the European Union: Seville, Spain, 2017. https://doi.org/10.2760/159770.
12. McCulloch, A.W.; Hollebrands, K.; Lee, H.; Harrison, T.; Mutlu, A. Factors That Influence Secondary Mathematics Teachers' Integration of Technology in Mathematics Lessons. *Comput. Educ.* **2018**, *123*, 26–40. https://doi.org/10.1016/j.compedu.2018.04.008.

13. Tatnall, A. Modeling the process of information technology innovation in education. In *Encyclopedia of Education and Information Technologies*; Springer: Cham, Switzerland, 2020; pp. 1200–1208.
14. Drijvers, P.; Ball, L.; Barzel, B.; Heid, M.K.; Cao, Y.; Maschietto, M. *Uses of Technology in Lower Secondary Mathematics Education: A Concise Topical Survey*; Kaiser, G., Ed.; ICME-13 Topical Surveys; Springer International Publishing: Cham, Switzerland, 2016; ISBN 978-3-319-33665-7.
15. Koehler, M.J.; Mishra, P.; Cain, W. What Is Technological Pedagogical Content Knowledge (TPACK)? *J. Educ.* **2013**, *193*, 13–19. https://doi.org/10.1177/002205741319300303.
16. Prediger, S.; Leuders, T.; Barzel, B.; Hussmann, S. Anknüpfen, erkunden, ordnen, vertiefen—Ein modell zur strukturierung von design und unterrichtshandeln. In *Beiträge zum Mathematikunterricht 2013: Vorträge auf der 47. Tagung für Didaktik der Mathematik*; Greefrath, G., Käpnick, F., Stein, M., Eds.; WTM Verlag: Münster, Germany, 2013; Volume 2, pp. 769–772.
17. Schoenfeld, A.H. When Good Teaching Leads to Bad Results: The Disasters of "Well-Taught" Mathematics Courses. *Educ. Psychol.* **1988**, *23*, 145–166. https://doi.org/10.1207/s15326985ep2302_5.
18. Engelbrecht, J.; Harding, A. Teaching Undergraduate Mathematic on the Internet. *Educ. Stud. Math.* **2005**, *58*, 235–252. https://doi.org/10.1007/s10649-005-6457-2.
19. Hoyles, C.; Noss, R. The Technological Mediation of Mathematics and Its Learning. *Hum. Dev.* **2009**, *52*, 129–147. https://doi.org/10.1159/000202730.
20. Pierce, R.; Stacey, K. Mapping Pedagogical Opportunities Provided by Mathematics Analysis Software. *Int. J. Comput. Math. Learn.* **2010**, *15*, 1–20. https://doi.org/10.1007/s10758-010-9158-6.
21. Bray, A.; Tangney, B. Technology Usage in Mathematics Education Research—A Systematic Review of Recent Trends. *Comput. Educ.* **2017**, *114*, 255–273. https://doi.org/10.1016/j.compedu.2017.07.004.
22. Puentedura, R.R. Transformation, Technology, and Education Available online: http://hippasus.com/resources/tte/ (accessed on 2 June 2021).
23. Boeskens, L.; Nusche, D. *Not Enough Hours in the Day: Policies That Shape Teachers' Use of Time*; OECD Education Working Papers; OECD Publishing: Paris, France, 2021; Volume 245;.
24. Haleem, A.; Javaid, M.; Qadri, M.A.; Suman, R. Understanding the Role of Digital Technologies in Education: A Review. *Sustainable Operations and Computers* **2022**, *3*, 275–285, doi:10.1016/j.susoc.2022.05.004.
25. Cavus, N.; Zabadi, T. A Comparison of Open Source Learning Management Systems. *Procedia Soc. Behav. Sci.* **2014**, *143*, 521–526. https://doi.org/10.1016/j.sbspro.2014.07.430.
26. Jones, K. Using Spreadsheets in the Teaching and Learning of Mathematics: A Research Bibliography. *MicroMath* **2005**, *21*, 30–31.
27. Niess, M.L.; van Zee, E.H.; Gillow-Wiles, H. Knowledge Growth in Teaching Mathematics/Science with Spreadsheets: Moving PCK to TPACK through Online Professional Development. *J. Digit. Learn. Teach. Educ.* **2010**, *27*, 42–52.
28. Juandi, D.; Kusumah, Y.S.; Tamur, M.; Perbowo, K.S.; Siagian, M.D.; Sulastri, R.; Negara, H.R.P. The Effectiveness of Dynamic Geometry Software Applications in Learning Mathematics: A Meta-Analysis Study. *Int. J. Interact. Mob. Technol.* **2021**, *15*, 18–37. https://doi.org/10.3991/ijim.v15i02.18853.
29. Leuders, T. Intelligent üben und Mathematik erleben. In *Mathemagische Momente*; Hefendehl-Hebeker, L., Weigand, H.-G., Eds.; Cornelsen: Berlin, Germany, 2009; pp. 130–143.
30. Hilgenheger, N. Johann Friedrich Herbart. *Q. Rev. Comp. Educ.* **1993**, *23*, 649–664.
31. Jones, A. Encouraging Creativity with Digital Technology in Early Primary Classrooms. *Aust. Educ. Comput.* **2004**, *19*, 8–11.
32. Freiman, V.; Tassell, J.L. Leveraging mathematics creativity by using technology: Questions, issues, solutions, and innovative paths. In *Creativity and Technology in Mathematics Education*; Freiman, V., Tassell, J.L., Eds.; Mathematics Education in the Digital Era; Springer International Publishing: Cham, Switzerland, 2018; Volume 10, pp. 3–29, ISBN 978-3-319-72379-2.
33. Joklitschke, J.; Rott, B.; Schindler, M. Notions, definitions, and components of mathematical creativity: An overview. In Proceedings of the 43rd Conference of the International Group for the PME, Pretoria, South Africa, 7–12 July 2019; Volume 2, pp. 440–447.
34. James, R. ICT's Participatory Potential in Higher Education Collaborations: Reality or Just Talk: ICT's Participatory Potential in HE Collaborations. *Br. J. Educ. Technol.* **2014**, *45*, 557–570. https://doi.org/10.1111/bjet.12060.
35. Olsher, S.; Thurm, D. The interplay between digital automatic- and self-assessment. In Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education, Khon Kaen, Thailand, 19–22 July 2021; Volume 3, pp. 357–364.
36. Hillmayr, D.; Ziernwald, L.; Reinhold, F.; Hofer, S.I.; Reiss, K.M. The Potential of Digital Tools to Enhance Mathematics and Science Learning in Secondary Schools: A Context-Specific Meta-Analysis. *Comput. Educ.* **2020**, *153*, 103897. https://doi.org/10.1016/j.compedu.2020.103897.
37. Volk, M.; Cotič, M.; Zajc, M.; Istenic Starcic, A. Tablet-Based Cross-Curricular Maths vs. Traditional Maths Classroom Practice for Higher-Order Learning Outcomes. *Comput. Educ.* **2017**, *114*, 1–23. https://doi.org/10.1016/j.compedu.2017.06.004.
38. Roschelle, J.; Singleton, C. Graphing calculators: Enhancing math learning for all students. In *International Handbook of Information Technology in Primary and Secondary Education*; Voogt, J., Knezek, G., Eds.; Springer US: Boston, MA, USA, 2008; Volume 20, pp. 951–959, ISBN 978-0-387-73314-2.

39. Sweller, J. Cognitive Load During Problem Solving: Effects on Learning. *Cogn. Sci.* **1988**, *12*, 257–285. https://doi.org/10.1207/s15516709cog1202_4.
40. Barzel, B.; Möller, R. About the Use of the TI-92 for an Open Learning Approach to Power Functions. *Zent. Didakt. Math.* **2001**, *33*, 1–5. https://doi.org/10.1007/BF02652764.
41. Gridos, P.; Avgerinos, E.; Mamona-Downs, J.; Vlachou, R. Geometrical Figure Apprehension, Construction of Auxiliary Lines, and Multiple Solutions in Problem Solving: Aspects of Mathematical Creativity. *Int. J. Sci. Math. Educ.* **2022**, *20*, 619–636. https://doi.org/10.1007/s10763-021-10155-4.
42. Peschek, W.; Schneider, E. CAS in General Mathematics Education. *Zent. Didakt. Math.* **2002**, *34*, 189–195. https://doi.org/10.1007/BF02655821.
43. Reinhold, F.; Hoch, S.; Werner, B.; Richter-Gebert, J.; Reiss, K. Learning Fractions with and without Educational Technology: What Matters for High-Achieving and Low-Achieving Students? *Learn. Instr.* **2020**, *65*, 101264. https://doi.org/10.1016/j.learninstruc.2019.101264.
44. Ziatdinov, R.; Valles, J.R. Synthesis of Modeling, Visualization, and Programming in GeoGebra as an Effective Approach for Teaching and Learning STEM Topics. *Mathematics* **2022**, *10*, 398. https://doi.org/10.3390/math10030398.
45. Kiru, E.W.; Doabler, C.T.; Sorrells, A.M.; Cooc, N.A. A Synthesis of Technology-Mediated Mathematics Interventions for Students with or at Risk for Mathematics Learning Disabilities. *J. Spec. Educ. Technol.* **2018**, *33*, 111–123. https://doi.org/10.1177/0162643417745835.
46. Oktaviane, D.A.K.; Ekawati, R. Development of Electronic Students' Worksheet Linear Function-Problem Based Using Desmos Application. *Cendekia* **2022**, *6*, 445–458. https://doi.org/10.31004/cendekia.v6i1.1153.
47. Lindenbauer, E.; Lavicza, Z. Using dynamic worksheets to support functional thinking in lower secondary school. In *Proceedings of CERME10*; Dooley, T., Gueudet, G., Eds.; DCU and ERME: Dublin, Ireland, 2017; ISBN 978-1-873769-73-7.
48. Radley-Gardner, O.; Beale, H.; Zimmermann, R. (Eds.) *Fundamental Texts on European Private Law*, 2nd ed.; Hart Publishing: Oxford, UK, 2016; ISBN 978-1-78225-867-4.
49. Al-zboon, H.S.; Gasaymeh, A.M.; Al-Rsa'i, M.S. The Attitudes of Science and Mathematics Teachers toward the Integration of Information and Communication Technology (ICT) in Their Educational Practice: The Application of the Unified Theory of Acceptance and Use of Technology (UTAUT). *World J. Educ.* **2021**, *11*, 75. https://doi.org/10.5430/wje.v11n1p75.
50. Birch, A.; Irvine, V. Preservice Teachers' Acceptance of ICT Integration in the Classroom: Applying the UTAUT Model. *Educ. Media Int.* **2009**, *46*, 295–315. https://doi.org/10.1080/09523980903387506.
51. Chao, C.-M. Factors Determining the Behavioral Intention to Use Mobile Learning: An Application and Extension of the UTAUT Model. *Front. Psychol.* **2019**, *10*, 1652. https://doi.org/10.3389/fpsyg.2019.01652.
52. Tabach, M.; Trgalová, J. Teaching mathematics in the digital era: Standards and beyond. In *STEM Teachers and Teaching in the Digital Era*; Ben-David Kolikant, Y., Martinovic, D., Milner-Bolotin, M., Eds.; Springer International Publishing: Cham, Switzerland, 2020; pp. 221–242, ISBN 978-3-030-29395-6.
53. Blömeke, S.; Gustafsson, J.-E.; Shavelson, R.J. Beyond Dichotomies: Competence Viewed as a Continuum. *Z. Psychol.* **2015**, *223*, 3–13. https://doi.org/10.1027/2151-2604/a000194.
54. Mayring, P. Qualitative content analysis: Theoretical background and procedures. In *Approaches to Qualitative Research in Mathematics Education: Examples of Methodology and Methods*; Bikner-Ahsbahs, A., Knipping, C., Presmeg, N., Eds.; Advances in Mathematics Education; Springer Netherlands: Dordrecht, Netherlands, 2015; pp. 365–380, ISBN 978-94-017-9181-6.
55. MAXQDA. How to Effectively Conduct Qualitative Research and Analyze Qualitative Data with MAXQDA Software. Available online: https://www.maxqda.com/blogpost/how-to-analyse-qualitative-data/ (accessed on 16 May 2022).
56. Coleman, T.D.; Walkoe, J.D.K. Productive Technology Use in Mathematics Explorations. *Math. Teach. Learn. Teach. PK-12* **2020**, *113*, 925–930. https://doi.org/10.5951/MTLT.2019.0137.
57. Cahyono, A.N.; Ludwig, M. Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia J. Math. Sci. Technol. Educ.* **2018**, *15*, em1654. https://doi.org/10.29333/ejmste/99514.
58. Kaspar, K.; Rott, B.; Bareth, G.; Becker-Mrotzek, M.; Großschedl, J.; Hofhues, S.; Hugger, K.-U.; Jost, J.; Knopp, M.; König, J.; et al. *Förderung digitalisierungsbezogener Kompetenzen von angehenden Lehrkräften im Projekt DiSK 2020;* Waxmann: Münster, Germany, 2020.
59. Makoe, M.; Thulile, S. Mobile learning and ubiquitous learning. In *Encyclopedia of Education and Information Technologies*; Springer: Singapore, 2020; pp. 1160–1169.
60. Davis, L.L.; Morrison, K.; Schnieders, J.Z.-Y.; Marsh, B. Developing Authentic Digital Math Assessments. *J. Appl. Test. Technol.* **2021**, *22*, 1–11.
61. Dalby, D.; Swan, M. Using Digital Technology to Enhance Formative Assessment in Mathematics Classrooms: Using Digital Technology in Formative Assessment. *Br. J. Educ. Technol.* **2019**, *50*, 832–845. https://doi.org/10.1111/bjet.12606.
62. Bailey, J.; Cowie, B.; Cooper, B. "Maths Outside of Maths": Pre-Service Teachers' Awareness of Mathematical and Statistical Thinking across Teachers' Professional Work. *Aust. J. Teach. Educ.* **2020**, *45*, 1–18. https://doi.org/10.14221/ajte.2020v45n1.1.
63. OECD. *Education at a Glance 2021: OECD Indicators*; Education at a Glance; OECD: Paris, France, 2021; ISBN 978-92-64-36077-8.
64. OECD. *Using Digital Technologies for Early Education during COVID-19: OECD Report for the G20 2020 Education Working Group*; OECD: Paris, France, 2021.

65. Fraillon, J.; Ainley, J.; Schulz, W.; Friedman, T.; Duckworth, D. *Preparing for Life in a Digital World: IEA International Computer and Information Literacy Study 2018 International Report*; Springer International Publishing: Cham, Switzerland, 2020; ISBN 978-3-030-38780-8.
66. Weinhandl, R.; Houghton, T.; Lindenbauer, E.; Mayerhofer, M.; Lavicza, Z.; Hohenwarter, M. Integrating Technologies into Teaching and Learning Mathematics at the Beginning of Secondary Education in Austria. *Eurasia J. Math. Sci. Tech. Ed.* **2021**, *17*, 1–15. https://doi.org/10.29333/ejmste/11428.
67. Soury-Lavergne, S.; Maschietto, M. Articulation of Spatial and Geometrical Knowledge in Problem Solving with Technology at Primary School. *ZDM Math. Educ.* **2015**, *47*, 435–449. https://doi.org/10.1007/s11858-015-0694-3.
68. Baccaglini-Frank, A.; Carotenuto, G.; Sinclair, N. Eliciting Preschoolers' Number Abilities Using Open, Multi-Touch Environments. *ZDM Math. Educ.* **2020**, *52*, 779–791. https://doi.org/10.1007/s11858-020-01144-y.
69. Sinclair, N.; de Freitas, E. Mathematical gestures: Multitouch technology and the indexical trace. In *Proceedings of CERME10*; Dooley, T., Gueudet, G., Eds.; DCU and ERME: Dublin, Ireland, 2017; ISBN 978-1-873769-73-7.
70. Papadakis, S.; Kalogiannakis, M.; Zaranis, N. Teaching Mathematics with Mobile Devices and the Realistic Mathematical Education (RME) Approach in Kindergarten. *Adv. Mobile Learn. Educ. Res.* **2021**, *1*, 5–18. https://doi.org/10.25082/AMLER.2021.01.002.
71. Hohenwarter, M. Multiple Representations and GeoGebra-Based Learning Environments. *UNIÓN* **2014**, *10*, 11–18.
72. Durner, A.; vom Orde, H. *International Data Youth and Media 2021*; International Central Institute for Youth and Educational Television (IZI): Munich, Germany, 2021.
73. Medienpädagogischer Forschungsverbund Südwest. *KIM-Studie 2020 Kindheit, Internet, Medien*; Landesanstalt für Kommunikation: Stuttgart, Germany, 2020.
74. UNICEF. *How Many Children and Young People Have Internet Access at Home? Estimating Digital Connectivity during the COVID-19 Pandemic*; UNICEF: Geneva, Switzerland, 2020; ISBN 978-92-806-5200-0.
75. UNICEF. *Prospects for Children in 2022*; UNICEF: New York, NY, USA, 2022.
76. Thurm, D.; Barzel, B. Teaching Mathematics with Technology: A Multidimensional Analysis of Teacher Beliefs. *Educ. Stud. Math.* **2022**, *109*, 41–63. https://doi.org/10.1007/s10649-021-10072-x.
77. Ministeriums für Schule und Bildung Nordrhein-Westfalen. *Lehrpläne für die Primarstufe in Nordrhein-Westfalen*; 1st ed.; Ministerium für Schule und Bildung des Landes Nordrhein-Westfalen: Düsseldorf, Germany, 2021; Vol. 2012; https://www.schulentwicklung.nrw.de/lehrplaene/ps_lp_sammelband_2021_08_02.pdf. (accessed on 3 Feb 2022).