Coding manual - "Dealing with student errors"

1 Introduction

This coding manual serves to analyse, as objectively as possible, how teachers deal with student errors in problem-solving lessons. The lesson is first divided into individual evaluation units. According to Mayring (2022, p. 60) these are the sequences that are analysed one after the other. The evaluation units are then assigned to the various coding categories, which are presented in Chapter 3.

2 Procedure for coding

In order to enable coding by different coders (interrater agreement), a detailed description of the coding procedure now follows: Firstly, an initial review of the material is carried out in order to obtain an overview of the data. The focus will initially be on the methodological structure of the lesson (introduction of the problem task and organisation of the lesson phases) as well as on incidents within the lesson. The rough course of the lesson is recorded, and conspicuous points are marked.

2.1 Determination of the units of analysis

After an initial review of the material to be analysed, a further review follows in which relevant units of analysis are determined. The units of analysis are sequences that consist of the teacher recognising an error situation and dealing with an error.

Start and end.
The beginning of such an analysis unit is characterised by the teacher encountering a learner or a group of students. This can take place on a verbal (e.g. by opening a conversation) or non-verbal level (e.g. by the teacher sitting next to students). The end of the analysis unit is characterised by one side of the conversation (i.e. the teacher or the students) leaving the conversation or changing the topic, which is not related to the reason for the conversation. If a new or further solution approach is addressed during a unit of analysis, a new unit of analysis is coded from this point onwards.

Interruption.
There may be interruptions to the error situation in individual analysis units. One possible reason for this is, for example, other students addressing the teacher. A unit of analysis is coded as continuous if the interruption lasts less than 30 seconds and no new error situation is revealed during the interruption.

2.2 Determining the cause of the error

Following the definition of the individual units of analysis, the solution approach or strategy chosen by the students and the error are determined. The determination of errors is based on the

taxonomy of errors in problem-based mathematics lessons according to Fritz (2022; based on Geering, 1995) is used as a guide. According to this taxonomy, strategy, skill, and knowledge errors are particularly relevant in problem solving.

Knowledge error
A knowledge error is coded when students do not use elements of knowledge that they know, such as mathematical theorems and definitions, or do not use them correctly. This is the case, for example, when students consider rectangles as well as squares in the chessboard task, although the definition of a square can be assumed to be already known.

Skill errors
Skill errors are coded when students do not correctly use basic and largely automated skills that they are already familiar with. An example of this is the incorrect calculation of multiplication problems in the number range up to 100.

Strategy error
Strategy errors are coded when students use unsuitable solution approaches or strategies for the problem task that lead to an incorrect result. This can be, for example, an incomplete determination of all possible combinations.

2.3 Coding of the handling categories

In the final step, the teaching unit is reviewed and checked again in order to assign the previously defined units of analysis to the handling categories in accordance with the coding manual (see chapter 3). All coding decisions should be recorded in writing. Any ambiguities must be discussed.

3 Categories

With the help of the categories listed, the ways in which teachers deal with problem-solving processes in the classroom can be analysed. In order to enable precise coding, in addition to a definition of the respective categories, the following provides anchor examples of a descriptive nature and coding rules that ensure clear categorisation and differentiation from other categories in individual cases. Based on Zech (1996) the categories are divided into 4 main categories: (1) Non-specific reaction, (2) Feedback aids, (3) Strategic aids and (4) Content aids.

Non-specific reaction
Unspecific reactions are those interactions of the teacher that occur directly in response to an error but have no relation to the task and the specific processing of the students.

Feedback aids

Zech (1996) describes feedback aids as those that provide the learner with information about whether he is right or wrong in his efforts to solve the problem (p. 316). No further information is provided.

Strategic assistance

In the case of strategic assistance, the problem solver is advised of (general or specific) heuristic strategies or other methods for further solving the task. Specific feedback on the meaningfulness of the previous solution process does not have to be provided.

Content aids

The last category is content-related aids. In this category, the teacher provides special information on predetermined terms and rules, on certain relationships between these, on very specific auxiliary variables or guidelines (Zech 1996, p. 317).

Remarks

It is to be expected that motivational aids can also be identified in further analyses. Only those are coded as motivational support in which the entire support is limited exclusively to a motivational aspect (cf. Zech 1996, p. 318). Overall, these are graded aids[1] . The highest form of help in the taxonomy is always coded.

[1] Unspecific reaction as the lowest form of help to Substantive help as the strongest form of help.

(1) Non-specific response (R_x)

Category	Definition of	Anchor example[2]	Coding rules
R_1 **Reception signal**	A reception signal is a signal from the listener that signals the person speaking to take part in the conversation without interrupting the other person's flow of speech. Typical reception signals are • Hm • Mhm • Aha • Yes This category is also coded when other emotional expressions such as laughing are identified as a reaction to a mistake.	*The teacher asks their students for their assessment of what result they expect for the chessboard task:* K_1 : 8 times 8 I think it is L: Hm, you mean. K_1 : 8 times 8 is 64, so that's 64 squares. L: Hm, hm Josha. K_2 : So I would make 64 squares, but the one on the outside, for example, is also a square or the one inside, or four squares together or something like that, so you can't say exactly (*specifically*?) L: Hm, Martha. [...] *(Miss G., school 1, task B3 - chessboard)*	This category is only coded if the only reaction to an error is a reception signal. If this is followed by another reaction, the corresponding category is coded.

[2] The excerpted transcripts were translated with the help of DeepL.

(2) General assistance (G_x)

Category	Definition of	Anchor example	Coding rules
G_1 **Correctness of result**	This category is coded if the teacher explains to the students that their result is incorrect.	*Student approaches the teacher and explains:* K_{18} : Miss P. ... I think I'm done, I have 118. L. No is more. K_{18} : What, great. *(Miss P., school 3, task B3 - chessboard)*	The category is only coded if no further help or information follows the feedback of the incorrect result. If further help follows, the corresponding category is coded.
G_2 **Reference to task conditions**	This category is coded when the teacher reminds the students to fulfil the conditions stated in the task. These may also be tacitly agreed tasks.	K_{20} : Can we also make the cake like this, with two pieces like this? L: How many tiers does the cake have? K_{20} : 3 L: Hm and one variety may occur how often. K_{20} : Once. L: Exactly, so that works Tilda' K_{20} : No. L: No. *(Miss B., School 2, Cake Combinatorics)*	

G_3 **Suggestive questions**	Suggestive questions are questions that are asked in such a way that a certain answer is particularly obvious. The answer is often already predetermined in these questions. If such a question is asked, this category is coded.	L: Can you bake a cake like that? K_{19} : (*shakes his head*) No. L: With only chocolate' K_{19} : (*laughs*) No. K_{20} : (*laughs*) No. L: No no. *(Miss B., School 2, Cake Combinatorics)*	
G_4 **Questions regarding the result**	This category is coded if the teacher draws the students' attention to an incorrect result by asking specific questions.	*The teacher stops at a group of students and asks for a number (45) that they have written down.* L: How much is how is the task now. K_3 : 8 times 8, so we calculated these first. L: How many are 8 times 8? K_3 : 64. *L: (laughs and leaves)* *(Miss G., school 1, task B3 - chessboard)*	
G_5 **Elimination of error**	This category is coded if the teacher makes a correction to the student's work by removing the error, for example by moving it, crossing it out, erasing it, etc. The	*K_{14} has drawn four solutions, where solutions 1, 3 and 4 are the same and differ in order. The teacher first explains why the solutions are the same.*	The category is characterised by an active action on the part of the teacher. If there is only a direct reference to the error and a request to correct the error,

	correction is an active action by the teacher, which is characterised by active intervention in the student's work.	L: (*Points to the third solution*) But that's the same (*crosses out the fourth solution with a pencil*) We'll do away with that one, because you already have it. A new possibility. Find another possibility. *(Miss T., school 4, coin task)*	category G_1 is coded as a direct explanation of the incorrect result.
G_6 **Request for control**	This category is coded if there is indirect error feedback in that the teacher indicates that a task, such as the result of a calculation, should be checked.	L: Look (*taps the card "7+8+9=17"*) please do the maths again here K_{19} : (*picks up the card*) *(Miss S., School 5, sequential numbers)*	The error is only referred to indirectly by the control request; a direct reference to the error by the teacher is coded as G_1.

(3) Strategic assistance (S_x)

Category	Definition of	Anchor example	Coding rules
S_1 **Sketches**	Sketches are graphic representations that are characterised by spatial-visual illustrations of objects - in the form of geometric objects, numbers, lines or similar. (Bräuer et al., 2021). This category is coded if the teacher recommends the	L: Hm, and then how can you be sure that the possibility doesn't occur twice? K_3 : And what if the colour is painted here? K_2 : Yes, you can do it that way. L: Maybe you could add that to your tally sheet, I think a tally sheet is a great idea, but somehow you would probably have to check that you don't have a double option or something.	

	problem solver to make a sketch or revise previous sketches in order to continue working on the task.	*The teacher stops with a group of pupils. They have found a way so far.* L: Maybe you could find a system in which you don't paint so elaborately because now you've already taken a very long time for one type, maybe you could find a way to paint it a bit faster so that you try to find all the possibilities with the different types. *(Miss B., School 2, Cake Combinatorics)*	
S_2 **Showing a counter-examples - with generalisation**	This category is coded if two conditions are met. Firstly, the teacher must show the problem solvers a part of the solution that has not yet been considered at this point. Secondly, the teacher must provide an additional indication of how the students can proceed in order to determine further aspects of the solution.	L: Hm, that means you have one here, one there, aha you took them like that ok, that's a good example[...] K_{13} : Hm. L: But what about this one, for example? K_{13} : Ts ts ts... ah. L: That means if I move it I can still find some, right? K_{13} : Oh good. *(Miss G., school 1, task B3 - chessboard)*	This category is only coded if both conditions are met. If only the demonstration of a partial solution that has not yet been considered is present, the category *C_1 – Showing a counterexample is* coded.

S_3 **Notation of solution path**	This category is coded if the teacher recommends the problem solver to write down their previous solution process.	*The students try to explain which squares they have found on the chessboard so far:* L: so write them down when you've written them down... it's not that easy (*laughs, 8 sec*) *(Miss G., school 1, task B3 - chessboard)*	
S_4 **Social exchange**	This category is coded if the teacher advises the problem solver to ask other students for solution tips or strategies. The teacher does not provide any other instructions for further processing of the problem task. The teacher can refer to the exchange with other students in general or specifically to individual students (e.g. seat neighbours, expert children, etc.).	*The students in a team share their solutions with their teacher. They have come to different conclusions:* L: But how can it be that he has 3 and you have 6 - maybe you could talk about it. K_{10} : He didn't use the same trick as me. Midfield used. L: But maybe you can talk about it, why don't you tell him the T the trick? K_{10} : Blue must be centre (*then incomprehensible*) L: Explain it to him again, maybe then he'll understand what else you've built. *(Miss B., School 2, Cake Combinatorics)*	

S_5 **Tip cards**	This category is coded if a teacher recommends that their students use tip cards or similar for further processing. No further instructions are given by the teacher.	K_{14} : We don't know what to do. L: How far are you? You don't know what to do. What tasks do you already have? K_4 : All these (*points to the cards*). L: That's great. You know what' (*points to the shelves*) You go to the back of the shelves and have a look at um tip 2. K_4 : Tip 2, good. L: There's a little card, you can take it here, look at it and then think about whether it helps you. *(Miss S., School 5, sequential numbers)*	

(4) Content-related assistance (C_x)

Category	Definition of	Anchor example	Coding rules
C_1 **Showing a counter-examples**	This category is coded when the teacher shows the problem solver a part of the solution that has not yet been considered. This makes it clear to the problem solvers that their previous solution is not yet complete.	K_3 : Erm, we have 92 squares out. L: Ahh... you're already super good you just have a little mental error like the others, for example, here you have yellow was these were these 4-squares ne' K_3 : Yes.	This category is only coded if no further information is given after the assistance has been pointed out. If, after pointing out the partial solution that has not yet been considered, a reference is made

		L: What about this one (*outlines a square on the pupil's chessboard*). $K_{3:}$ Mmhm. L: (*goes to the next group table*) *(Miss G., school 1, task B3 - chessboard)*	to a further possible procedure, the category *S_2 – Showing a counterexample – with generalisation* is coded.
C_2 **Providing a solution approach**	This category is coded if the teacher specifies part of the solution through their statements or actions. This specification (partially) removes the problem barrier, and the task becomes a routine task.	*After two students explain that they do not know how many small squares there are (by pointing, it becomes clear that the individual squares are meant), the teacher asks the students to count the squares in the row and then in the column and corrects the count several times. The teacher then asks the students to write down the result and explains that they now must count the 2x2 squares (points to one on the chessboard with their finger). The number is determined under the guidance of the teacher. This is repeated for the 4x4 squares. Then the teacher leaves the situation.* *(Miss P., school 3, task B3 - chessboard)*	

4 Bibliography

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